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STUDY GUIDE:

PHYSICS HL

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IB Academy Physics Study Guide

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INTRODUCTION

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MEASUREMENTS AND MATHEMATICAL FOUNDATIONS

1

1.1 Physical measurements

1.1.1 Fundamental and derived SI units

Fundamental units

There are six fundamental SI units from which all other units can be derived.

Time	seconds	s
Displacement	meters	m
Mass	kilograms	kg
Temperature	kelvin	K
Amount of substance	mole	mol
Current	ampere	A

Derived units

All other units can be expressed as a combination of these fundamental units, therefore derived from them.

Example.

Since speed $= \dfrac{\text{displacement}}{\text{time}}$, or $v = \dfrac{s}{t}$, the units for v are

$$\frac{\text{unit of displacement}}{\text{unit of time}} = \frac{[\text{m}]}{[\text{s}]} = \text{m}\,\text{s}^{-1}$$

In a similar manner we can derive other units:

- Force = Mass × Acceleration = $[\text{kg}] \times [\text{m}\,\text{s}^{-2}]$, therefore the unit is $\text{kg}\,\text{m}\,\text{s}^{-2}$.

  $1\,\text{kg}\,\text{m}\,\text{s}^{-2} = 1\,\text{N}$, or newton.
- Pressure $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{[\text{kg}\,\text{m}\,\text{s}^{-2}]}{[\text{m}^2]}$, therefore the unit is $\text{kg}\,\text{m}^{-1}\,\text{s}^{-2}$.

  $1\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-2} = 1\,\text{Pa}$, or pascal.
- Energy = Force × Distance, therefore the unit is $\text{kg}\,\text{m}^2\,\text{s}^{-2}$.

  $1\,\text{kg}\,\text{m}^2\,\text{s}^{-2} = 1\,\text{J}$, or joule.

Exercize.

Write the compound units as a combination of SI units

Compound quantity		Calculation	Derived unit
Power	[watt]	$P = \frac{E}{t}$	________________
Charge	[coulomb]	$Q = A \times t$	________________
Resistance	[ohm)	$R = \frac{V}{I}$	________________
Magnetic field strength magnetic flux density	[tesla]	$B = \frac{F}{IL}$	________________

1.1.2 Orders of magnitude

Order of magnitude a method of comparing the sizes of values, with each order of magnitude being equivalent to a multiple of 10.

Metric multipliers prefixes that precede units to indicate its order of magnitude.

As an example, the order of magnitude of 1500 is 3, and in scientific notation it will be written as 1.5×10^3. Meanwhile, 0.1 is 2 orders of magnitude smaller than 10, since it is 100 times smaller than 10.

One can also use prefixes to indicate the order of magnitude of a unit. So, 1000 m is 1 km, while 2×10^{-3} J is equal to 1 mJ. The SI prefixes used by IB can be seen in Table 1.1.

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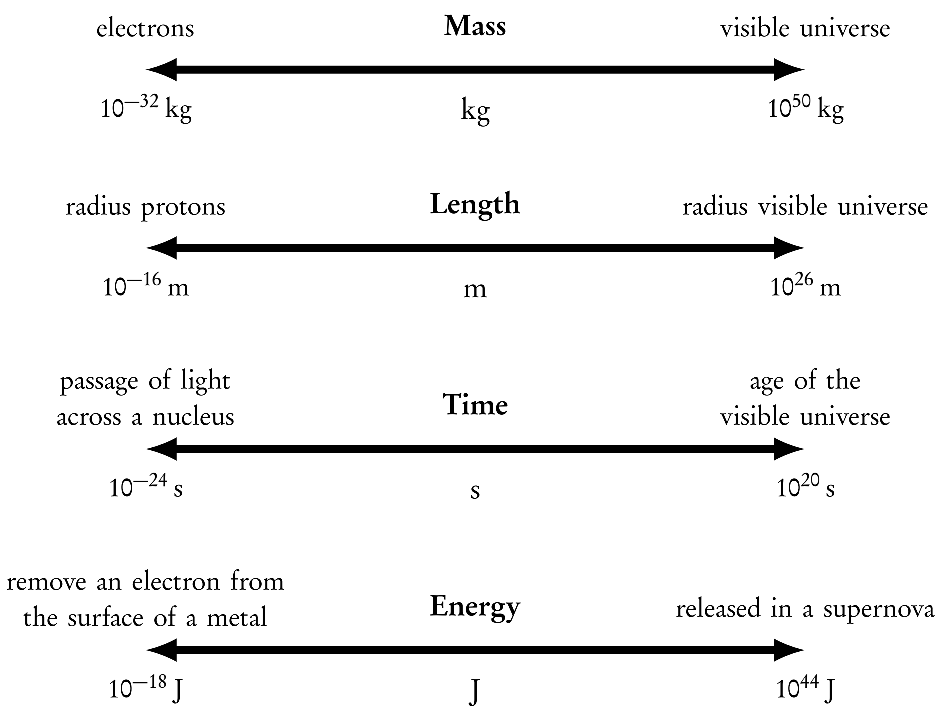
Table 1.1: Metric (SI) multipliers

Prefix	Symbol	Power	Prefix	Symbol	Power
yocto	y	10^{-24}	deca	da	10^{1}
zepto	z	10^{-21}	hecto	h	10^{2}
atto	a	10^{-18}	kilo	k	10^{3}
femto	f	10^{-15}	mega	M	10^{6}
pico	p	10^{-12}	giga	G	10^{9}
nano	n	10^{-9}	tera	T	10^{12}
micro	µ	10^{-6}	peta	P	10^{15}
milli	m	10^{-3}	exa	E	10^{18}
centi	c	10^{-2}	zetta	Z	10^{21}
deci	d	10^{-1}	yotta	Y	10^{24}

1.1.3 Estimation of quantities

To show your understanding of both the SI units and the orders of magnitude, IB often tests your knowledge of these in a paper 1 exam, relating values dealing with atoms to astronomical objects and events.

It is important to have a general awareness what scales such objects deal with.



1.1.4 Significant figures

Significant figures are used to show the accuracy of a measurement.

Determination of the number of significant figures can be summarised with the following rules.

1. Non zero digits are significant, e.g. 4.643 has 4 significant figures: $\overset{1}{4}.\overset{2}{6}\overset{3}{4}\overset{4}{3}$
2. Zero digits between non-zero digits are significant, e.g. 809 has 3 significant figures: $\overset{1}{8}\overset{2}{0}\overset{3}{9}$
3. Zero digits after a decimal are significant if they lie to the right of a non-zero digit, e.g. 4.000 and 0.03200 both have 4 significant figures: $\overset{1}{4}.\overset{2}{0}\overset{3}{0}\overset{4}{0}$ $0.0\overset{1}{3}\overset{2}{2}\overset{3}{0}\overset{4}{0}$
4. All other zero digits are *not* significant, e.g. 0030 has 1 significant figure: $00\overset{1}{3}0$

To ensure you get your significant figures correct, make sure you look at your calculations with the two definitions shown below.

1. When multiplying or dividing, check which given value used in your calculations has the least number of significant figures.
   - This should be the number of significant figures in your answer, e.g. $1.34 \times 4.8 = 6.432$ should be expressed as 6.4 because 4.8 has only 2 significant figures.
2. When adding or subtracting, check which given value used in your calculations has the least number of decimal places.
   - This should be the number of decimal places your answer, e.g. $7.34 + 4.8 = 12.14$ should be expressed as 12.1 because 4.8 has only 1 decimal place, even though this exceeds the number of significant figures in 4.8.
3. If one of the given values comes from a graph:
   - Use the amount of significant figures corresponding to the smallest grid on the graph, e.g. for a graph grid units of 0.1, a reading such as 3.65 should be expressed to only 2 significant figures, as 3.6.

Exercize.

Determine the number of significant figures in the following values

1. 102
2. 0.00235
3. 2.0×10^{-2}
4. 314.159

1.2 Uncertainties and errors

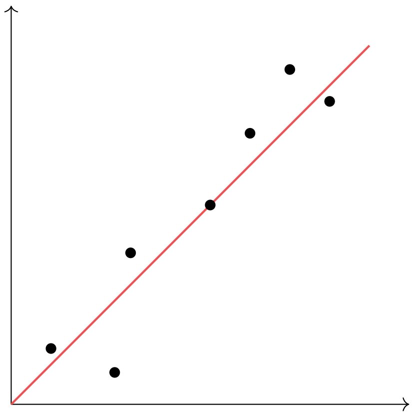
Errors have always been a part of experiments as you've likely encountered in practicals. It is important to deal with this in calculations, which can be done using a set of simple rules explained in the next few sections.

1.2.1 Random and systematic errors

These types of errors should not be confused with simple mistakes such as misreading an instrument, writing down the wrong number or making a calculation error, which are not considered sources of experimental error.

Random errors

Affects each measurement in a random manner.



Leads to a less *precise* experiment.

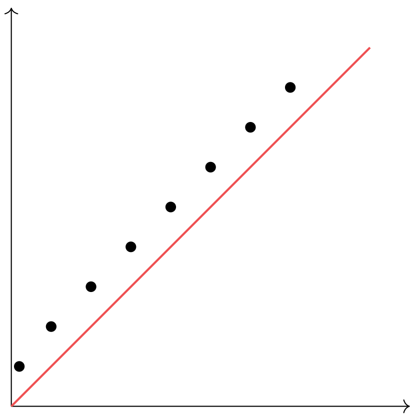
Caused by fluctionations in the instrument readings, observer interpretation and effects due to changes in the surroundings.

Reduced by repeated readings.

Data points are spread around the perfect data, the more measurements made the closer the data matches the perfect results.

Systematic errors

Affects each measurement in the same way.



Lead to a less *accurate* experiment.

Caused by wrongly calibrated apparatus and imperfect methods of observation.

Reduced by ensuring instruments are properly calibrated, by mathematically removing known offsets, or by changing the way a measurement is taken. Data is either proportional to the perfect results, or deviating by some constant value. By checking the y-intercept the size of the error can be determined.

1.2.2 Absolute, fractional, and percentage uncertainties

Turn off your intuitive perception on uncertainties and allow your mathematical side to come to life. When approached with a simple mathematical formula, calculating the uncertainty will become a routine procedure. Moreover, it will also help your understanding of the uncertainty of the compound or calculated values.

Absolute uncertainty the uncertainty in a measurement as an absolute value, e.g. ± 0.5

Fractional (relative) uncertainty the uncertainty in a measurement as a fraction of the measurement, given by $\dfrac{\text{absolute uncertainty}}{\text{measurement}}$.

Percentage uncertainty the fractional uncertainty in a measurement expressed as a percentage, given by fractional uncertainty $\times$ 100%, e.g. $\pm 0.5\%$

The following rules are given in the data booklet and are all the manipulations you will need to know for your exams.

Note that when expressing a measurement, use only an absolute or percentage uncertainty (to avoid a fractional uncertainty being confused for an absolute uncertainty). Fractional uncertainties are used in intermediate calculations.

DB St 1.2 (p4)

Addition and subtraction: $y = a + b$ and $y = a - b$

The uncertainty after addition/subtraction is the *sum* of the *absolute uncertainties* of a and b.

$$\Delta y = \Delta a + \Delta b$$

Multiplication and division: $y = \dfrac{ab}{c}$

The uncertainty after multiplication/division is the *sum* of the *fractional uncertainties* of a, b and c.

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

where $\dfrac{\Delta y}{y} = $ % fractional uncertainty.

Exponential: $y = a^n$

The uncertainty after an exponent n is the *multiplication* of the *fractional uncertainty* by the power n.

$$\frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$$

Example.

If the absolute uncertainty is defined as 1 cm when measuring your height to be 2 m then the percentage uncertainty is 0.5%.

$$\frac{1\,\text{cm}}{200\,\text{cm}} \times 100\% = 0.5\%$$

Exercize.

If the absolute uncertainty is 8 cm when measuring Kim's height to be 160 cm then the fractional uncertainty is $\frac{8}{160} \times 100 = \frac{1}{20} \times 100$ which correponds to a percentage uncertainty of 5%.

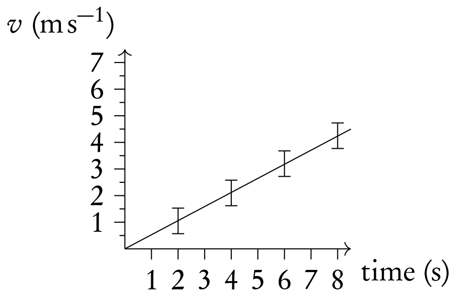
1. What would the fractional uncertainty of Bill's measurement be if he is measured at 200 cm with an absolute uncertainty of 8 cm?
2. If Kim were to stand on Bill's head what would you estimate their combined height to be?

Often, the question will ask you to find the absolute uncertainty in the solution, Δy. Typically you are given absolute uncertainties but sometimes the IB can be tricky and give you the percentage uncertainty, so pay close attention.

Exercize.

Uncertainties from Graphs

Samir accelerates constantly and his velocity v increases according to the graph below. The accelerometer used isn't perfect and this is represented by the error bars.



If Samir exerts an average force $F = (1000 \pm 100)\,\text{N}$ per stop, how much power does Samir generate at $t = 4\,\text{s}$? Use $P = Fv$.

Exercize.

Let $a = 2 \pm 0.5$ and $b = 5 \pm 1.2$.

1. Calculate the uncertainty in $c = a + b$.
2. Calculate the uncertainty in $c = ab$.

ACADEMY

1.2.3 Graphs

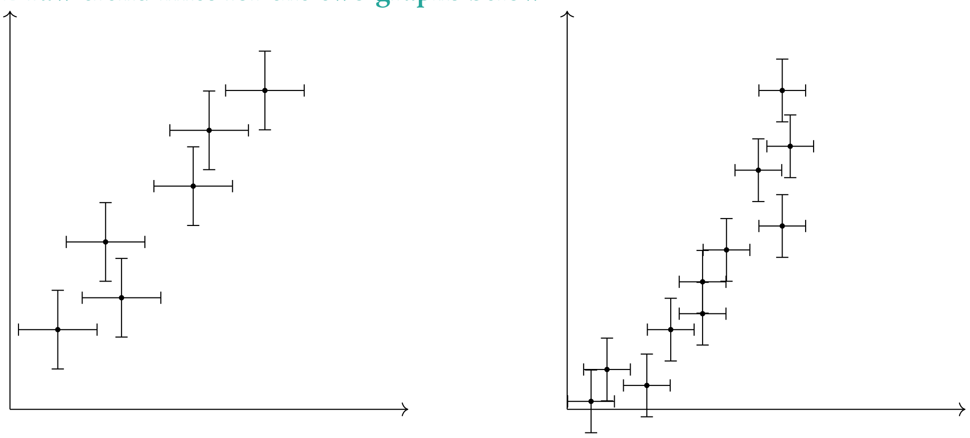
Drawing a trend line

Questions regarding uncertainty may also be shown graphically using error bars. A typical question the IB may ask is to draw a line of best-fit. When drawing this line, you should *always* keep these very important rules in mind.

1. Identify whether the points follow a linear or non-linear progression.
2. *Always!* Start at the origin.
3. Draw your trend-line ensuring you go through all error bars.

Exercize.

Draw trend-lines for the two graphs below



Finding the gradient of a line

The gradient of a straight-line graph represents how fast the dependent quantitiy changes in relation to the independent quantity, e.g. a gradient of 2 means that for every 1 unit on the x axis, the y value changes by 2.

The gradient can be determined by picking two points on the line (as far apart as possible, to reduce error). For two points (x_1, y_1) and (x_2, y_2), the gradient m is given by

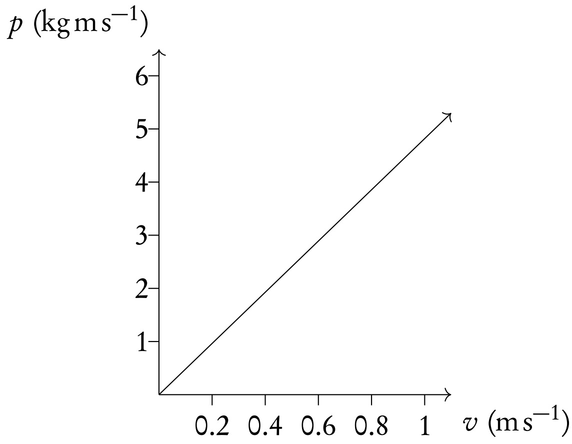
$$m = \frac{y_2 - y_1}{x_2 - x_2}$$

The point where the line cuts the y axis is known as the intercept and is usually denoted c.

Thus we can write the equation of the line $y = mx + c$, which links the dependent variable to the independent variable.

Exercize.

Find the gradient



What does the gradient represent in this example?

1.3 Vectors

Usually, the image you get when you hear the word vector is an arrow pointing in a certain direction. To picture the arrow you need to know its direction and its length. In physics, we assign a physical meaning to the length of a vector, for example the magnitude of a force. In this way, the vector can be used to represent a force of a particular magnitude operating in a particular direction. In the IB, you will be using vectors in nearly every topic and you will see that they actually make life much easier by allowing you to combine quantities that would be tedious to work with as components. Vectors also allow you to quickly draw a diagram of almost any physical situation and thus develop some intuition regarding what is going on.

1.3.1 Vectors and scalars

A vector is defined as a quantity which has both a magnitude and a direction.

A scalar is defined as a quantity which has only a magnitude *without* any associated direction.

For example, speed is a quantity that has a magnitude and units ($\mathrm{m\,s^{-1}}$) but has no direction associated with it. Its vector counterpart is velocity, which does have a direction.

Exercize.

Name three vector quantities and three scalar quantities.

1.3.2 Combination and resolution of vectors

While vectors are similar to scalars in that you can add and subtract them, the exact procedure by which this is done is somewhat different since you need to take the vector's direction into account.

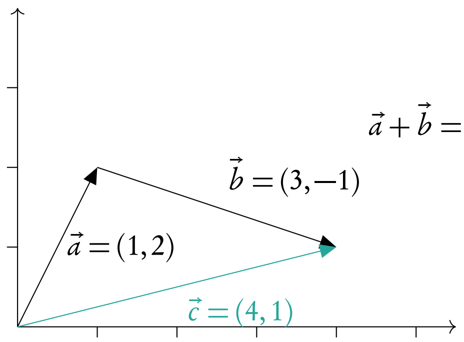
The techniques outlined in this section are all the necessary ones for vector calculations in IB physics.

Example.

Mathematical vector manipulation

Addition and subtraction of vectors

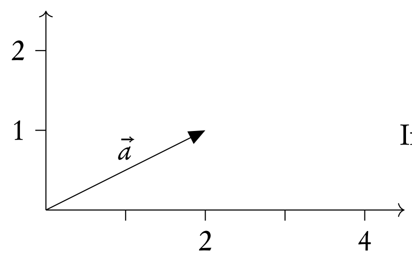
Imagine there are two displacement vectors, $\vec{a}$ and $\vec{b}$. We can find their sum if we connect the tail of the one to the point of the other. It does not matter in which order we add the vectors. Note if you go along the opposite direction of a vector its positive values are negative and vice versa.



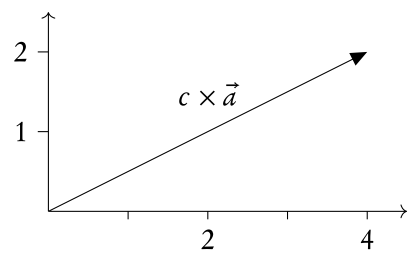
$$\vec{a} + \vec{b} = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}$$

Multiplication and division of vectors by scalars

If I have a vector $\vec{a}$ and multiply it with a scalar c, then the result is simply $c \cdot \vec{a}$, i.e.:



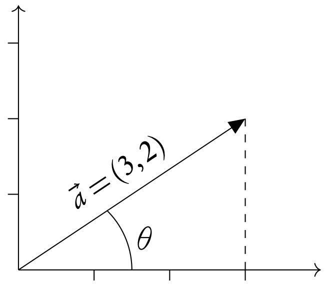
If $c = 2$, then:



Example.

Trigonometric form

Especially in topics like mechanics, there will be cases where you'll need to split a vector into two parts, generally into a vertical and horizontal part. To do this you need the magnitude of your vector and the angle θ it makes with the x-axis. Your data booklet contains a diagram that shows how to find the component form of a vector



$$\hat{x} = 3 \qquad \hat{y} = 2$$

$$\begin{aligned} |\vec{a}| &= \left(x^2 + y^2\right)^{\frac{1}{2}} \\ &= (9+4)^{\frac{1}{2}} = \sqrt{13} \\ \theta &= \arctan\left(\frac{y}{x}\right) \\ &= \arctan\left(\frac{2}{3}\right) = 33.7° \end{aligned}$$

MECHANICS

2

2.1 Motion

We want to describe how bodies move through space by considering their displacement, velocity and acceleration over periods of time. Keep in mind that all of these are vector quantities and so we need to keep their direction in mind!

2.1.1 Displacement, velocity, acceleration & the equations of motion

Displacement a vector quantity stating the distance removed from a reference point.

Velocity a vector quantity stating the rate of change of the displacement.

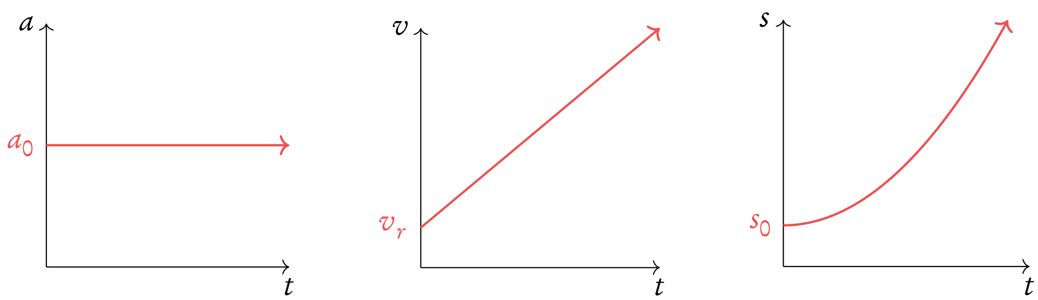
Acceleration a vector quantity stating the rate of change of the velocity.

Within the IB, we will always approach problems under the assumption of uniform motion. If this is not the case, the IB will explicitly state this in the beginning of a problem!

Uniform motion the motion of a body under constant (possibly zero) acceleration.

Typically, you will be asked to find instantaneous values for velocity, speed, and acceleration either from graphs or using the *equations of motion*.

DB The equations of motion are:



$$a = a_0 \qquad v = v_r + at \qquad s = s_0 + v_r t + \frac{at^2}{2}$$

Aside from instantaneous velocity, the IB will at times ask you to find and use the average velocity. Given that there is constant acceleration the average velocity is given by:

$$\vec{v}_{av} = \frac{\vec{u}+\vec{v}}{2} \qquad \left(\begin{array}{l} \vec{u} = \text{initial velocity} \\ \vec{v} = \text{final velocity} \end{array} \right)$$

In the IB exams, occasions will arise where both the observer and the target are moving, which means that we are dealing with relative velocity. In this case, the velocity $\vec{v}$ of the target is the sum of the observer's velocity $\vec{v}_o$ and the targets velocity with respect to the observer $\vec{v}_r$.

$$\vec{v} = \vec{v}_o + \vec{v}_r$$

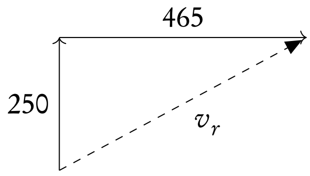
Relative velocity between two bodies is the velocity of one body in the rest frame of the other.

Example.

Imagine you're flying upwards at $v_f = 250\,\text{m}\,\text{s}^{-1}$ perpendicular to the Earth's rotation, with the Earth spinning at $v_E = 465\,\text{m}\,\text{s}^{-1}$ to the right.

An astronaut at the ISS is measuring how fast you're going.

What's your relative speed, v_r?
First we draw the two velocity vectors and place them end to end like below.



Since the two vectors form a right triangle, we can find the v_r by Pythagoras' theorem.

$$\begin{aligned} v_r &= \sqrt{(250)^2 + (465)^2} \\ &= \sqrt{62500 + 216225} \\ &\approx 528\,\text{m}\,\text{s}^{-1} \end{aligned}$$

What is the direction θ of the velocity with respect to the Earth's rotation?
To find the angle θ, we use that $\tan(\theta) = \frac{v_f}{v_E}$. Therefore, the angle θ is:

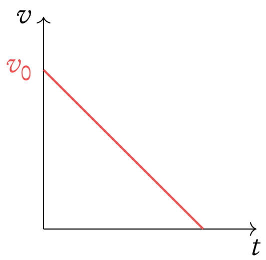
$$\theta = \arctan\left(\frac{250}{465}\right) \approx 28°$$

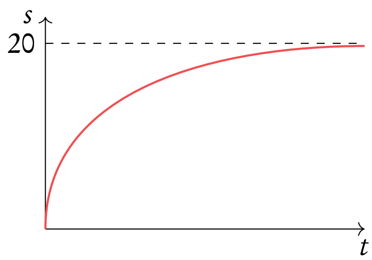
2.1.2 Graphs describing motion

For the following graphs, you will be given various scenarios. Draw the corresponding displacement and velocity curves on the axes given. The first one has been done for you as an example.

Example.

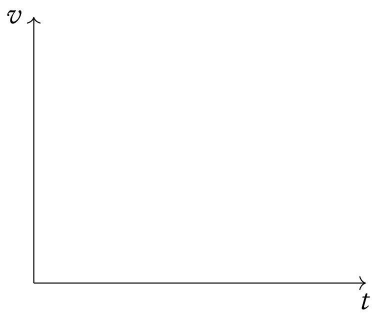
A car is coming to rest at constant deceleration from velocity v_0, it takes the car 20 m to come to a halt from $S_0 = 0$.

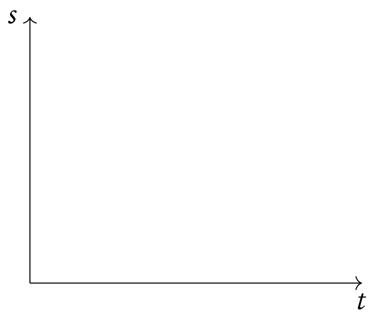




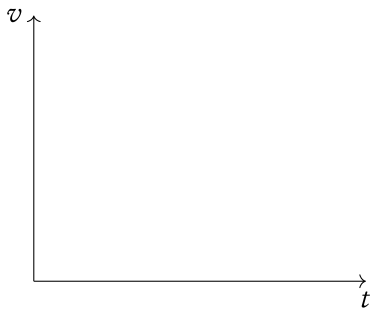
Exercize.

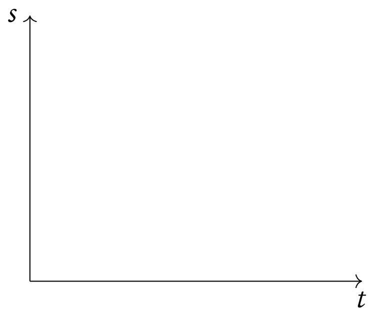
A bullet fired at a wall at constant velocity v_0. The bullet travels 50 m before it hits the wall.





A ball rolls down a hill and then up another, coming to rest at the top of the hill. Assume acceleration from gravity.





2.1.3 Projectile motion

Projectile motion deals with objects as they fly through the air (often bullets and balls). Often you will be given the initial velocity in a certain direction, then it is a simple matter of breaking the velocity into x and y components. Using the vector component diagram from your data booklet, a velocity with magnitude v_0 and direction θ, has the velocity components:

$$\begin{cases} v_x = v_0 \cos(\theta) \\ v_y = v_0 \sin(\theta) \end{cases}$$

You can now solve for them independently.

x-component

As we are assuming zero air resistance, it follows that in the x-direction, there are no external forces acting on the projectile. Therefore we use the equation:

$$s_x = s_{x,0} + v_x t \tag{2.1}$$

where s_x is displacement, $s_{x,0}$ is initial displacement and v_x is velocity in this direction.

Typically, the initial displacement will be given as zero, the velocity in the x-direction can be found using equation (2.1), but first we must find the unknown variable of time, t. To find this point in time, we move to the y-component.

y-component

When regarding the y-component, we still neglect air resistance. However we cannot forget that gravity is acting on the projectile.

It follows that we must use the equation:

$$s_y = s_{y,0} + v_y t + \frac{g t^2}{2}$$

In general, we can use this to solve for time. This will be done with slightly initial values depending on the problem but in the IB you will *always* use that.

Steps to consider when solving a projectile motion problem

1. The projectile returns to initial displacement in the y-direction $s_{y,0}$ after a time t.
2. The projectile will be at its maximum displacement in the y-direction s_{max-y} after a time $\frac{t}{2}$. Furthermore, at this point it will also be at the midpoint of its parabolic path.

Example.

A typical problem you might find in the IB about projectile motion.

A centaur shoots an arrow at a 30° angle with an initial velocity of $v_0 = 50\,\mathrm{m\,s^{-1}}$, initial displacement $= 0$.

How long does it take for the arrow to hit the ground?

Regard y-component:

$$S_y = S_{y,0} + v_{y,0}t + \frac{gt^2}{2}$$

Use final conditions, at $t = t_{\max} \Rightarrow S_y = S_{y,0}$, thus:

$$\begin{aligned} S_{y,0} &= S_{y,0} + v_{y,0}t_{\max} + \frac{g t_{\max}^2}{2} \\ -v_{y,0}t_{\max} &= \frac{g t_{\max}^2}{2} \\ v_{y,0} &= v_0 \sin\theta = 50\sin(30) = 25 \\ 25 &= \frac{10}{2}t_{\max} \\ t_{\max} &= 5\,\mathrm{s} \end{aligned}$$

What's the maximum height the arrow reaches?

Maximum height is at $\frac{t_{\max}}{2} = 2.5\,\mathrm{s}$, thus:

$$\begin{aligned} S_{y,\max} &= S_{y,0} + v_{y,0}\left(\frac{t_{\max}}{2}\right) + \frac{g\left(\frac{t_{\max}}{2}\right)^2}{2} \\ S_{y,\max} &= 0 + (25)(2.5) + \frac{10(2.5)^2}{2} = 94\,\mathrm{m} \end{aligned}$$

How far does the arrow fly?

Total distance is calculated by:

$$\begin{aligned} S_x &= S_{x,0} + v_{x,0}t_{\max} \\ v_{x,0} &= v_0\cos\theta \\ &= (50)(\frac{\sqrt{3}}{2}) = 25\sqrt{3} \\ S_x &= (25\sqrt{3})(5) \\ S_x &= 125\sqrt{3} \approx 215.5 \end{aligned}$$

2.1.4 Terminal velocity

To conclude motion, we will spend a little time talking about the consequences of fluid resistance. As mentioned in the beginning, in the IB all problems are approached under the assumption of negligible air resistance. However, sometimes you will be asked to explain in words or pictures what happens if we do take it into account.

Terminal velocity the constant speed that a freely falling object eventually reaches when the resistance of the medium through which it is falling prevents further acceleration.

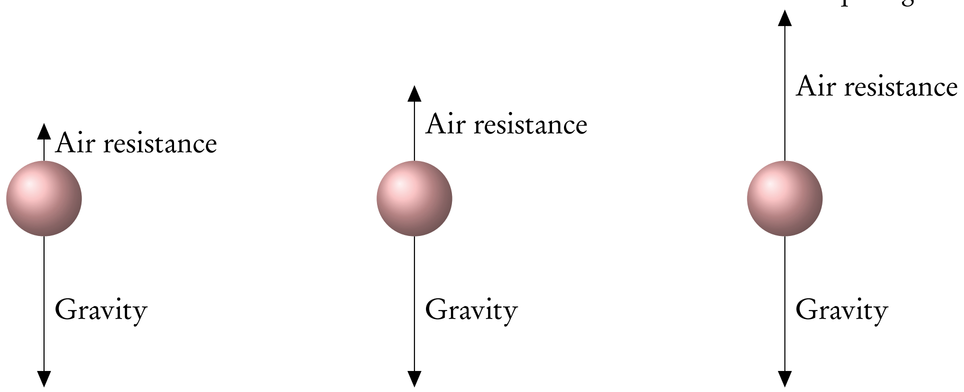
To get a better understanding, we will regard the situation of an object dropping from an arbitrary height towards earth. When the object is first dropped, it has zero velocity, as it is not moving there is also no resistance force. However, as the object starts dropping it starts moving faster.

Air resistance is an exponential consequence of motion, as the object starts moving faster, it will experience a greater resistance force. At a certain point, the object will stop increasing in velocity and continue falling at *terminal velocity*. At this point, the air resistance is so great that it is equal to the force of gravity, but in the opposite direction! It follows that the net force on the object is zero and thus it will no longer increase in velocity.

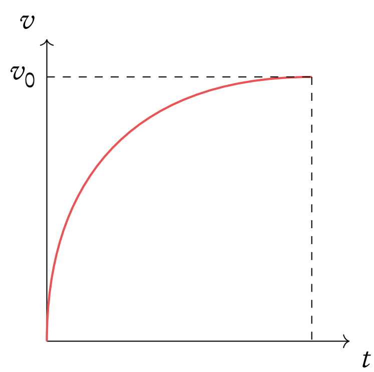
Ball starts accelerating due to gravity.

Ball continues accelerating, air resistance increases.

Ball reaches terminal velocity, when air resistance equals gravity.



If we plot the change in velocity over time, we get something that looks like this:



2.2 Forces

Force is the vector quantity most often used in IB physics. Typically, we regard the object in question as a point particle. This assumption basically allows one to take the object as a whole and not as a rigid body. We've already seen an example of an object being regarded as a point particle within the context of a force diagram when looking at terminal velocity in the previous section.

In mechanics, we will only be considering two forces, deformation and changes in velocity.

A force is a push or a pull that causes a change in magnitude *and/or* direction of velocity.

Point particle is an object represented as a point, ignoring that it is in fact a rigid body.

2.2.1 Newton's laws of motion

Although forces can be interpreted rather intuitively, we should follow a certain set of rules to describe such a fundamental concept.

Fortunately for us, these sets of rules have been well established by Isaac Newton in 1687.

Newton's 1st Law If the force F on an object is zero then its velocity v is constant.

$$F_{\text{net}} = 0 \Rightarrow \frac{\Delta v}{\Delta t} = 0$$

Newton's 2nd Law The force F of an object is equal to its mass m times its acceleration a.

$$F = ma$$

F	= force	[N]
m	= mass	[kg]
a	= acceleration	[m s^{-2}]

Newton's 3rd Law Every action has an equal but opposite reaction.

$$\vec{F}_a = -\vec{F}_b$$

2.2.2 Solid friction

Friction arises whenever one body slides over another, or whenever there is a tendency for motion.

Dynamic friction

The force of dynamic friction is equal to:

$$F_d = M_d R$$

R	= normal reaction force	[N]
M_d	= coefficient of dynamic friction	dimensionless

Note that the force of dynamic friction does not depend on the speed of sliding.

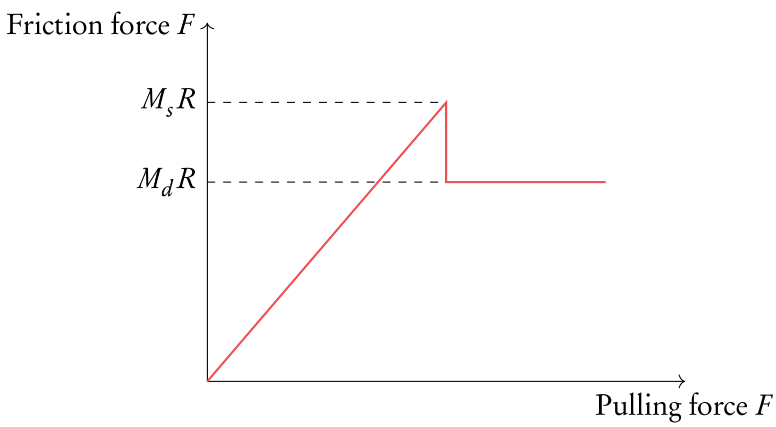
Static friction

The maximum force of static friction that can develop between two surfaces is given by:

$$F_s = M_s R$$

R	= normal reaction force	[N]
M_d	= coefficient of static friction	dimensionless

Note that $M_s > M_d$.



2.3 Work, energy, and power

Out of all the various concepts in physics, energy is the most universal. It is present in many different forms and can also transform between its various forms.

In mechanics, we regard three forms of energy:

1. Kinetic energy
2. Gravitational potential energy
3. Elastic potential energy

As we move to other topics, we will encounter other forms of energy and how we transform between them.

2.3.1 Kinetic, gravitational, and elastic potential energy & the principle of conservation of energy

We will always be regarding transformations between kinetic and potential energy. Whether the potential energy is stored as gravitational or elastic is arbitrary.

DB

In equation form, the three types of energy may be written as:

kinetic	$E_k = \frac{1}{2}mv^2$
gravitational	$E_g = mgh$
elastic	$E_e = \frac{1}{2}kx^2$

Principle of conservation of energy states that in any closed system, total energy will always be conserved or stay constant.

When regarding transformations in energy, we may assume that in a closed system, the total energy is conserved.

Exercize.

Conservation of energy

Imagine the situation where Bryan and Lucy are standing on the same cliff and want to jump into the water at the same time. If Bryan jumps as high as he can while Lucy dives straight down, assuming they jump at the same initial speed, who will hit the water with a greater velocity?

If Bryan and Lucy both weigh 60 kg they jump at 5 m/s.

1. What is the maximum height Bryan will reach if the cliff is 10 m above the water?
2. With what velocity will each of them hit the water?

2.3.2 Work done as energy transfer

Work is the amount of force exerted in the direction of motion.

$$W = Fs\cos\theta$$

W	= work	[J]
F	= force	[N]
s	= direction	[m]

θ is the angle between the force and the direction of motion.
Work is also defined as the total change of energy of an object.

Exercize.

Look at the problem of Bryan and Lucy again, how much work does Bryan exert against the force of gravity if the average force until he reaches the maximum height is 150 N?

Hint: Assume $\theta = 0$ and find s, the distance from the top of the cliff to the maximum height.

Besides regarding work done in the context of force and distance, it is useful to look at work in the context of energy transformations. This change in energy can be gravitational potential energy as in the case of Bryan jumping off the cliff. However, it can also be in the form of kinetic energy, if you exert force over a certain distance, the object will remain in motion if no external forces act upon it. We will come back to work done, defined as the change in energy, in both thermal physics and electrical circuits.

Work done may be defined as a change in energy:

$$W = E_2 - E_1 = \Delta E$$

Example.

For example, if I move a 5 kg box onto a shelf, 2 m from the floor, the work done will be:

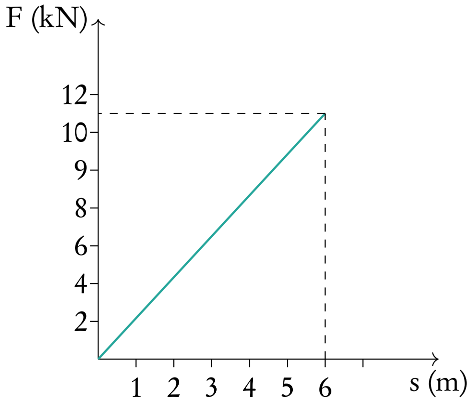
$$W = mgh_2 - mgh = (5)(10)(2-0) = 100\,\text{J}$$

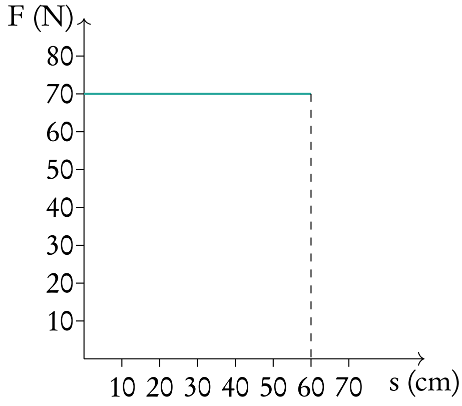
Exercize.

Assuming a frictionless surface, how much work will it take to get a 10 kg ball initially resting to move at $v = 5\,\text{m/s}$?

Exercize.

Sometimes the IB will ask you to sketch and interpret force-distance graphs. Look at the graphs below and answer the associated questions.





How much work was done on the object shown in the graphs above?

2.3.3 Power as rate of energy transfer & efficiency

Power is typically defined as the energy change over a certain period of time.

$$P = \frac{\Delta E}{\Delta t}$$

$P =$ power $[\text{J s}^{-1}]$
$E =$ energy $[\text{J}]$
$t =$ time $[\text{s}]$

As the change in energy may be defined as work done, we can also write power as:

$$P = \frac{W}{\Delta t} = \frac{F \Delta s \cos\theta}{\Delta t} = F v \cos\theta$$

Efficiency is the ratio of useful power coming out of a system to the total power going into the system

$$\text{Efficiency} = \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total work in}}$$

2.4 Momentum and impulse

2.4.1 Newton's 2 law & force-time graphs

Momentum is what gives the objects their force. One of the most important laws of physics is conservation of momentum; we will see this later in application to collisions.

Momentum is what is used to describe the motion of massive bodies.

Momentum can be defined by re-writing Newton's 2nd law:

$$F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$$

Impulse is the change in momentum of an object.

$$\Delta p = F\Delta t$$

Δp	= impulse	[N s]
F	= force	[N]
t	= time	[s]

You will often be asked to find the value of Δp from an F vs t diagram.

The law of conservation of momentum says that in any closed system, momentum is always conserved, in other words, momentum is conserved if $F_{ext} = 0$.

It is good to keep these definitions in the back of your mind, however you will be using the equation below this more often in your calculations.

$$\vec{p} = m\vec{v}$$

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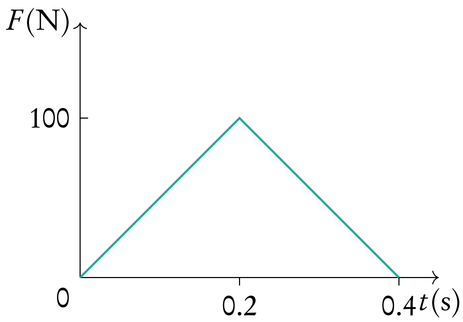
Exercize.

Often, problems will ask you about impulse. Here is a problem about someone pushing an object for a very short amount of time, a perfect example of impulse.



What's the maximum force applied to the object?

What's the average force applied to the object?

What's the total change in momentum after the push?

We now return to the concepts of conservation of momentum with the use of collisions as an example.

Elastic collisions energy is conserved.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Inelastic collisions energy isn't conserved.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

THERMAL PHYSICS

3

3.1 Thermal concepts

3.1.1 Temperature, heat energy, and internal energy

Upon measuring how hot an object is, we are assuming a dynamic equilibrium in the flow of heat. That is, the net flow of heat between two systems is equal to zero.

Temperature measure of how hot something is (it can be used to work out the direction of the natural flow of thermal energy between two objects in thermal contact) or measure of the average kinetic energy of molecules. It is measured on a defined scale (celsius, kelvin)

$$K = {}^{\circ}C + 273$$

Heat energy transferred from one body to another due to a temperature difference. It is measured in joule (J).

Remember: heat is the transfer of thermal energy. *Heating is a process*, not *a property* (like temperature, pressure, etc.).

Internal energy the sum of all random kinetic energies and mutual potential energies of the particles of the body or system. It is measured in joule (J).

Internal energy does not include the kinetic energy or potential energy of the body as a whole.

An ideal gas has no intermolecular forces, therefore the gas particles have no mutual potential energies therefore the internal energy of an ideal gas depends only on the kinetic energy of the particles (temperature of gas).

Kinetic and potential energies are defined into more specific subgroups of motion and forces.

Kinetic Energy	Potential Energy
Vibrations	Lattice forces
Rotations	Intermolecular forces
Translations	

Upon heating or cooling the system, one of two things may happen.

- A change in kinetic energy will change the temperature of the system.
- A change in potential energy will change the phase of the system.

3.1.2 Avogadro & the mole

The mole is used to quantify large amounts of molecules. As we use a dozen to say we have 12 of something, we have a specific variable to say that we have 6.02×10^{23} of something, that is Avogadro's number, $N_A = 6.02 \times 10^{23}$.

Avogadro constant, N_A the number of atoms in exactly 12×10^{-3} kg of the nuclide carbon-12.

Mole amount of substance of a system which contains as many elementary units as there are carbon atoms in 12×10^{-3} kg of carbon-12. The mole is the SI unit for measuring the amount of a substance.

Molar mass the mass of one mole of a substance.

3.2 Thermal properties of matter

3.2.1 Thermal capacity & specific heat capacity

Whenever internal energy is spent on increasing the kinetic energy of a system, the temperature of the system will change. On a microscopic level, a change in temperature is simply an increase in average kinetic energy of the particles that make up the system.

Specific heat capacity, c the amount of energy required to raise the temperature of a unit of mass through 1 K.

Heat (thermal) capacity, C the amount of energy/heat required to raise the temperature of a substance/object through 1 K.

DB

Specific heat capacity

$$Q = mc\Delta T$$
$$Q = C\Delta T$$

Q	= heat or amount of energy added	[J]
m	= mass	[kg]
c	= specific heat capacity	[J kg^{-1} K^{-1}]
C	= thermal capacity	[J K^{-1}]
ΔT	= change in temperature	[K]

Typically, we will be regarding objects or systems with a known mass. In this case, we will have to regard the specific heat capacity.

3.2.2 Phases of matter

To gain a little more intuition into why these phases are so different from one another, we will take a quick look at each of them on a microscopic level.

Phase	Macroscopic	Microscopic
Solid	Volume and shape are approx. constant	Particles held by bonds • Vibrations
Liquid	Volume is approx. constant, shape varies	Weak intermolecular (IM) forces • Translational • Vibrational
Gas	Varying shape and volume	Random motion • Translational • Vibrational

3.2.3 Phase changes

Intermolecular bonds are bonds that hold molecules together due toe forces of attraction.

Phase changes involve the breaking or loosening of intermolecular bonds. When the phase of a body changes, only the potential energy changes, kinetic energy and thus temperature will stay constant. When bonds break, the potential energy of the body increases, as energy is absorbed, and vice versa.

3.2.4 Latent heat

Latent heat is very similar to specific heat capacity except we are not looking at the heat required to change the temperature of a system by a certain amount. Instead we want to know how much heat is needed to change the phase of a certain mass of substance (i.e. melting from solid to liquid).

Specific latent heat is defined as the amount of heat required to change the state of 1kg of substance.

Specific latent heat DB

$$Q = mL$$

Q = heat [J]
m = mass [kg]
L = specific latent heat [$\mathrm{J\,kg^{-1}}$]

3.3 Kinetic model of an ideal gas

3.3.1 Assumptions of the Kinetic Model

There are a few assumptions that should be remembered when dealing with ideal gases; these assumptions can be very easily remembered using the acronym, NITES.

- **N** → Newton's law of mechanics apply
- **I** → Intermolecular forces are negligible
- **T** → Time of collisions is negligible
- **E** → Elastic collisions (no energy lost)
- **S** → Small spherical/point particles

The IB will usually only ask for three assumptions, so if you remember this acronym you have a very high chance of getting full points on any relevant question.

In an ideal gas, temperature is directly proportional to the average kinetic energy of the system.

$$T \propto \frac{mv_{\text{avg}}^2}{2}$$

3.3.2 Pressure

Pressure can be looked at in two regimes, the macroscopic and the microscopic regimes.

Macroscopically, pressure is defined as the force exerted over a unit area.

Pressure

$$p = \frac{F}{A}$$

P = pressure [N m^{-2}] or [Pa]
F = force [N]
A = area [m^2]

Microscopically, we consider pressure as the sum of counterforces of all colliding particles with the container over a certain area and between particles themselves.

This involves Newton's third law, as the container in which a gas is held will push back on the particles with an equal and opposite force, enforcing the pressure. If the force of the particles becomes too great for the container to hold, the container will break, as a balloon might if pumped up too large.

3.3.3 State equation of an ideal gas

There is one equation that is very common and useful in thermal physics, the state equation of an ideal gas. A real gas may be approximated by an ideal gas when the density is low. Remember that density is simply $\frac{n}{V}$.

State equation of an ideal gas DB

$$nR = \frac{PV}{T}$$

n =	amount of substance	[mol]
R =	ideal gas constant	$8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$
P =	pressure	[Pa]
V =	volume	[$\mathrm{m^3}$]
T =	temperature	[K]

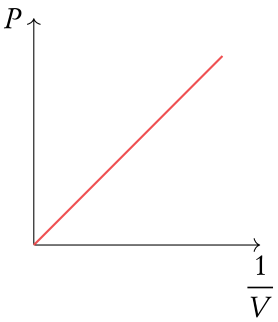
This equation is a direct consequence of the assumptions presented above and the way we have defined pressure. It allows us to relate the various thermodynamic quantities to each other. Expect to be given several of the quantities and making use of this equation in order to calculate the remaining value.

The state equation may be separated into individual proportionality laws that will assist your understanding. These may all be derived from the ideal gas law, when we assume a closed container so that nR does not change.

$$\frac{P_1 V_1}{T_1} = nR = \frac{P_2 V_2}{T_2}$$

At constant temperature $T_1 = T_2$, so that the above becomes $P_1 V_1 = P_2 V_2$.

The pressure-volume law

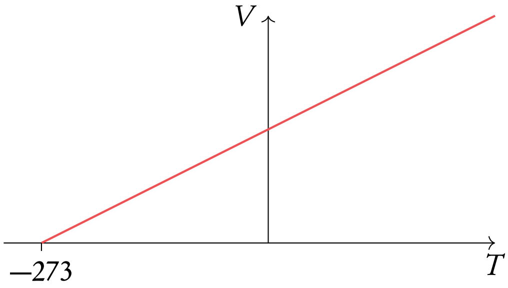


At constant temperature, pressure and volume display inverse proportionality as shown in the diagram.

$$P \propto \frac{1}{V} \qquad \text{or} \qquad PV = \text{constant}$$

This is also known as *Boyle's law*.

The volume-temperature law



When measured experimentally in kelvin at constant P, volume and temperature show a linear relationship as as shown in the diagram.

$$\frac{V}{T} = \text{constant}$$

This is also known as *Charles' law*.

Pressure-temperature relationship

This was similarly found for the pressure-temperature relationship.

$$\frac{P}{T} = \text{constant}$$

This is known as the *Gay-Lussac's law*.

OSCILLATIONS & WAVES

4

IB. ACADEMY

4.1 Kinematics of simple harmonic motion

Displacement (x) The amount of distance that an oscillation may be found from equilibrium at an arbitrary point in time.

Amplitude (A) The maximum value the oscillation can have.

Period (T) The amount of time it takes to complete a full oscillation.

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Simple harmonic motion (SHM) is oscillatory motion where the resistance force or acceleration is negatively proportional to the displacement.

Conditions of simple harmonic motion:

- Oscillations around an equilibrium point
- Sinusoidal relations between position and time

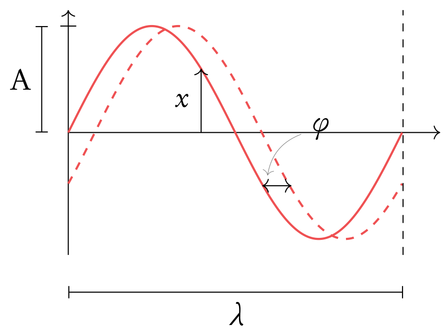
Wavelength (λ) the length of a full period of oscillation.

Angular frequency (ω) The amount of full oscillations per second.

$$\omega = 2\pi f$$

Phase difference (φ) The difference in period between two points in an oscillation

DB



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SHM equations:

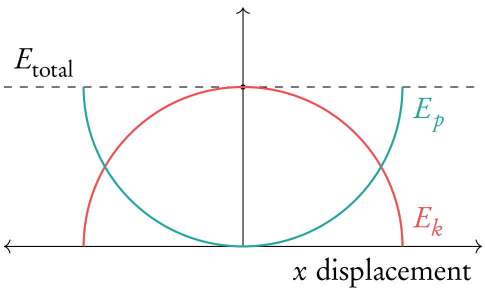
$$F_{res} = -kx \qquad ma = -kx \qquad a \propto -x$$

The solution to this relationship is what we will use in order to solve various problems regarding displacement, velocity and acceleration.
Solutions:

$$x = A\sin(\omega t)$$
$$v = a\omega\cos(\omega t)$$
$$a = -A\omega^2\sin(\omega t)$$

4.2 Energy changes during SHM

Conservation of energy holds in simple harmonic motion. This means that the total energy stays constant and we again have a constant exchange between potential and kinetic energy. It is simpler in oscillations because the total energy in the system is easily defined using the maximum amplitude of the oscillation. The following graph shows the interchange between kinetic and potential energy in the system.



These graphs are governed by the following equations depicting the energy changes that occur during an oscillation.

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$ DB

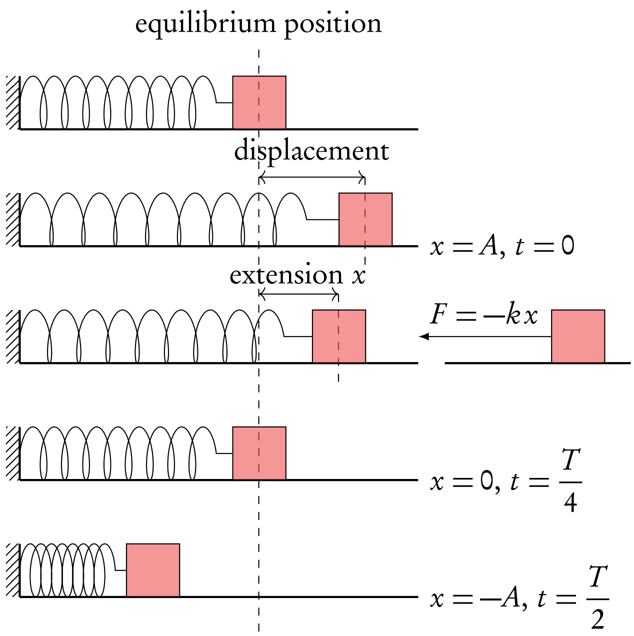
$$E_p = \frac{1}{2}m\omega^2 x^2$$

$$E_{tot} = E_k + E_p = \frac{1}{2}m\omega^2 A^2$$

Simple Harmonic Motion(SHM) is described by: $a = -\omega^2 x$. ω is the angular frequency and is related to the period by: $T = \frac{2\pi}{\omega}$ DB

There are two conditions for SHM. Take the mass-spring system as an example.

Example.



From this we can deduce that: DB

$$\omega^2 = \frac{k}{m}$$

thus,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

At small amplitudes the simple pendulum may be approximated as undergoing SHM.

$$\omega^2 = \frac{g}{L}$$

We can calculate velocity at any displacement using: $v = \pm\omega\sqrt{x_0^2 x^2}$. DB

ACADEMY

4.3 Forced oscillations & resonance

4.3.1 Damping

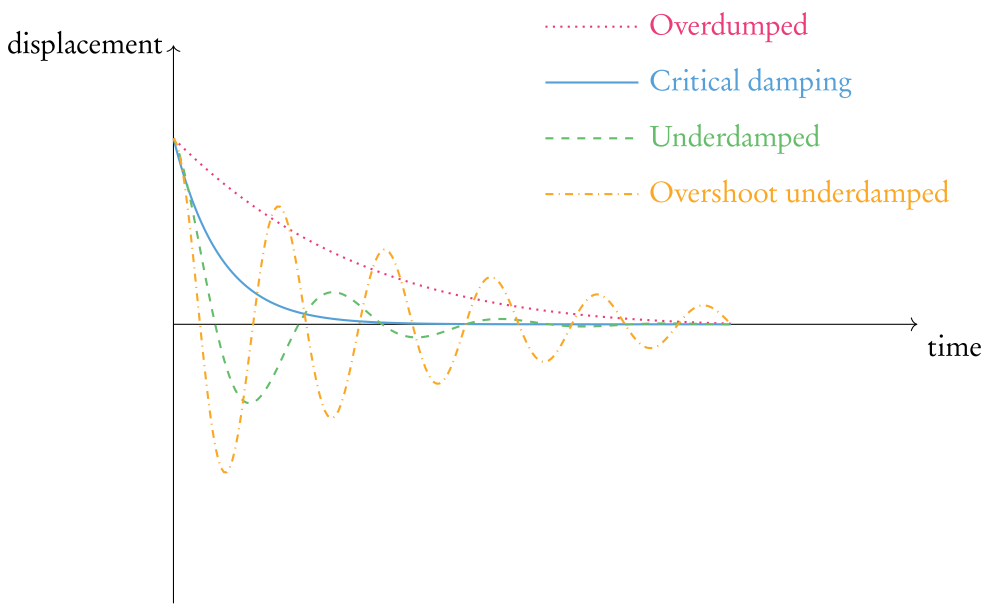
Damping the process whereby friction causes oscillations to tend back to the equilibrium in real oscillators.

Underdamped there is very little friction; the oscillation goes to zero after several oscillations.

Overdamped a lot of friction; the oscillation never even breaches the equilibrium point and goes slowly to zero without oscillating.

Critically damped this is the optimal amount of friction. For example, car springs are damped so as to go to zero after 1 or 2 oscillations.

Figure 4.1: Levels of damping



4.3.2 Resonance

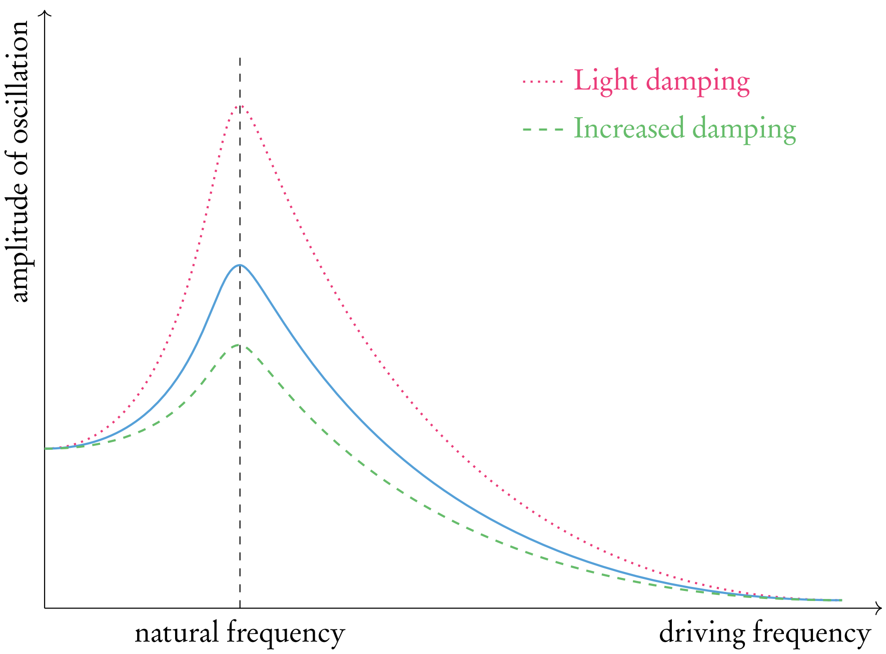
The application of an external force may either reinforce or obstruct oscillatory motion. Imagine a father pushing his kid on a swing, if he doesn't time the push exactly as the kid is moving away from him, he will obstruct the oscillation of the swing. This is because every oscillation has a natural frequency associated with it.

Resonance is a phenomenon that the system experiences when the driving frequency approaches the natural frequency.

Natural frequency is defined by the physical limitations of the system.

Driving frequency is the frequency of the external force.

Figure 4.2: Resonance



As the frequency of the external periodic force approaches the natural frequency of oscillation, the effect of resonance on the amplitude of oscillation will increase.

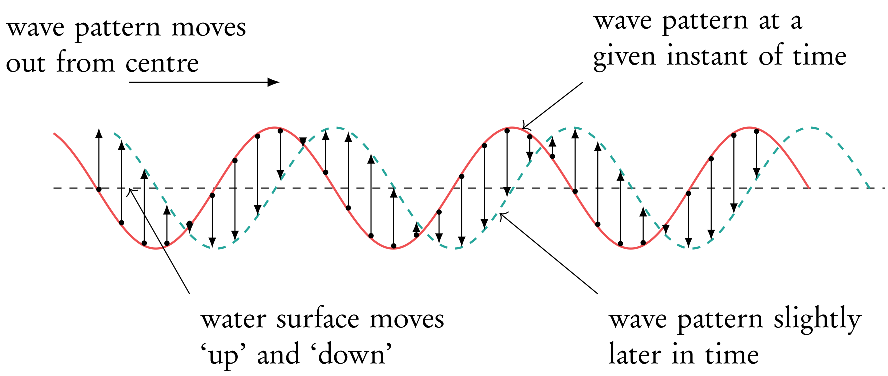
4.4 Wave characteristics

4.4.1 Travelling waves

Travelling waves move in space by using the medium they are travelling through to propagate energy. The medium itself is NOT moving in the direction of the wave, it simply oscillates in place due to the energy of the travelling wave.

Transverse waves

Transverse waves are travelling waves in which the direction of oscillation is perpendicular to the direction of energy propagation.



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The direction of the wave velocity is the same as the propagation of energy, the wave velocity is given by:

$$v = f\lambda = \left(\frac{s}{t}\right)$$

Longitudinal waves

Longitudinal waves are travelling waves in which the direction of oscillation is parallel to the direction of energy propagation.

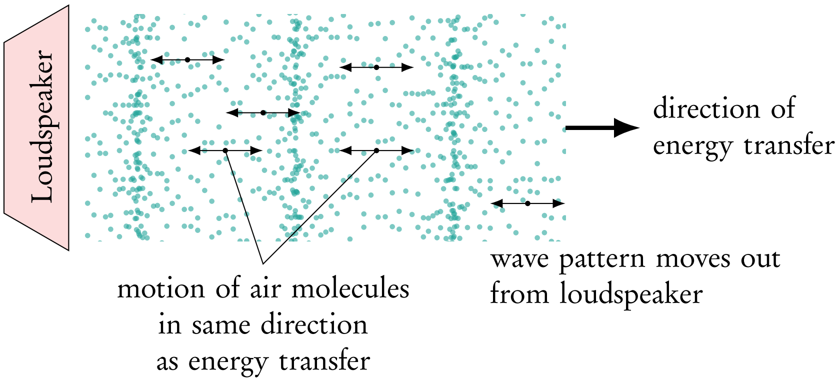
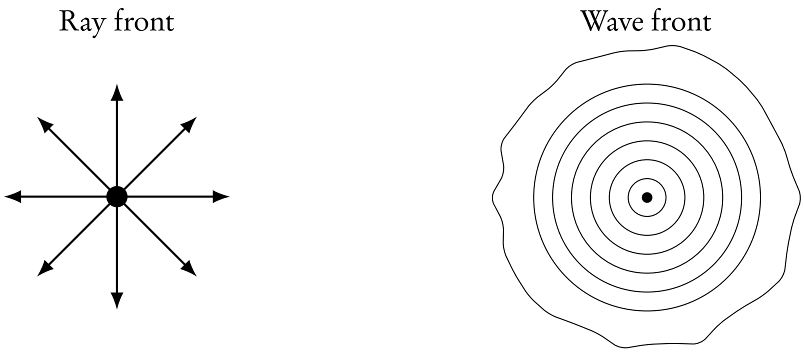
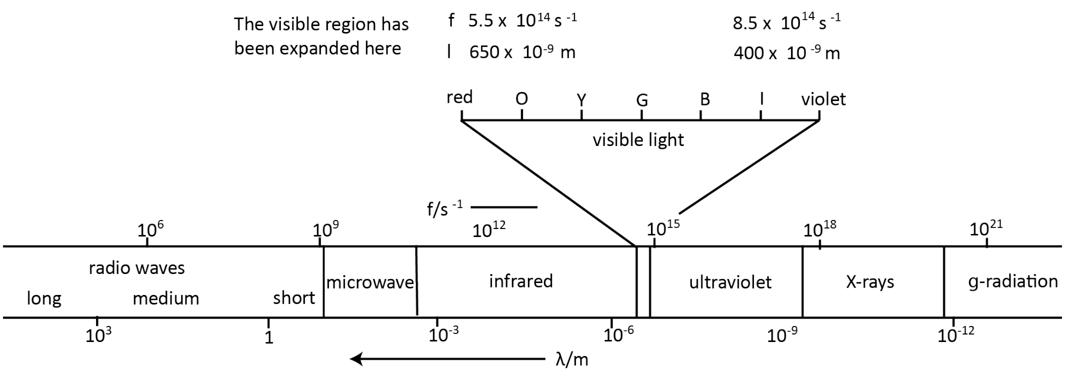


Figure 4.3: Direction of energy flow.



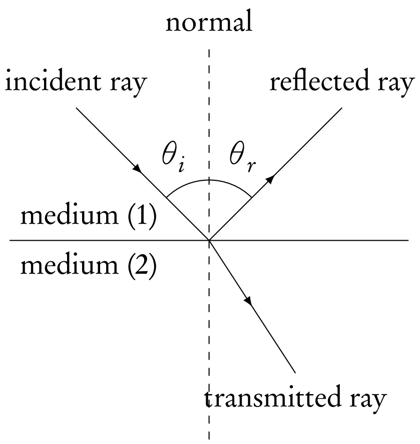
4.4.2 Electromagnetic waves

Electromagnetic waves are unique in that they don't need a medium in order to propagate energy. They propagate energy supported by electric and magnetic fields. In the next figure you can see the various important frequency / wavelength / energy ranges of EM waves.



4.5 Wave properties

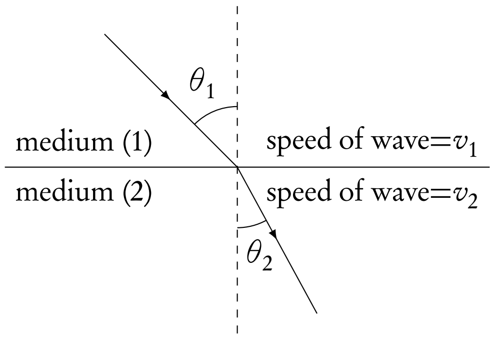
Reflection



Imagine for example, a ray of light being reflected off water or a mirror. This phenomenon is rather intuitive in that the incident angle θ_i is equal to the angle of reflection θ_r.

$$\theta_i = \theta_r$$

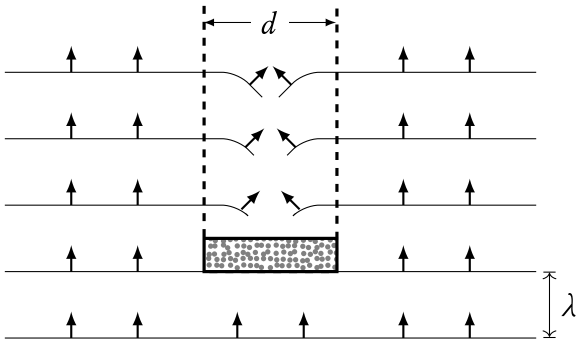
Refraction



Important to remember is that as a ray enters a medium with a different refractive index, the wavelength and velocity will change and the frequency will remain constant.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

Diffraction



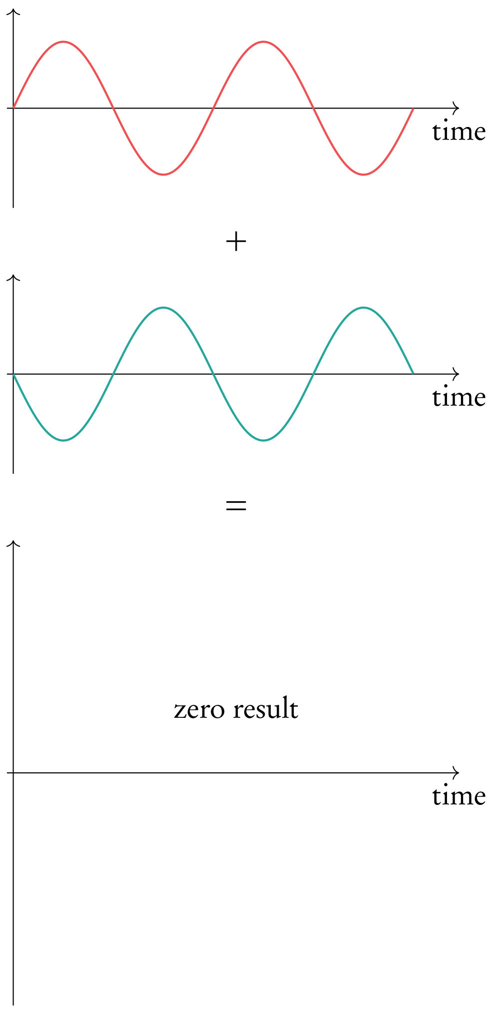
For diffraction to occur, the disturbance must be of the *same* order of magnitude as the wavelength. Furthermore, as the wave is diffracted, the wavelength does *not* change, nor does the frequency or velocity.

Interference

In all problems encountered within the IB, the sources of the waves are coherent, this means that they have the same wavelength. When the waves add up, they undergo constructive interference; when they subtract, they undergo destructive interference.

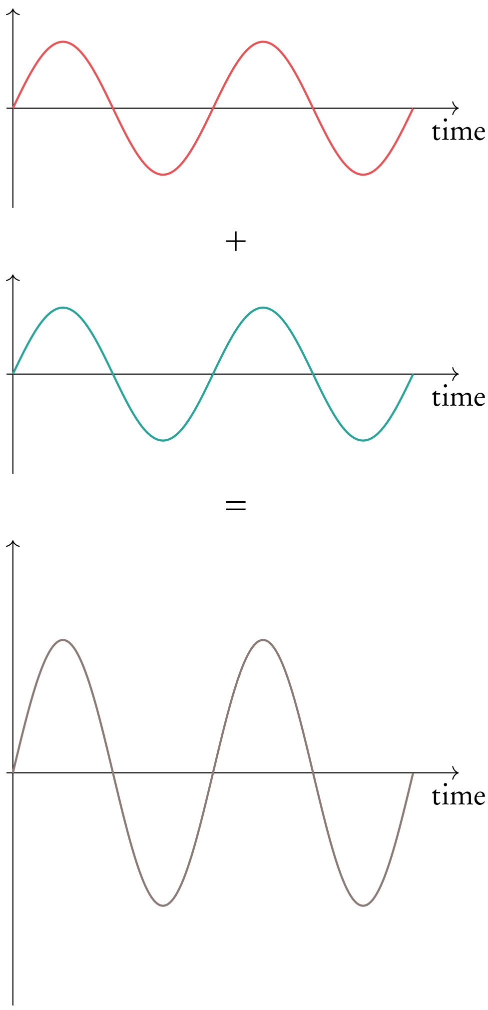
Destructive interference

$$\left(n+\frac{1}{2}\right)\lambda = \text{path difference}$$



Constructive interference

$$n\lambda = \text{path difference}$$



4.6 Standing waves

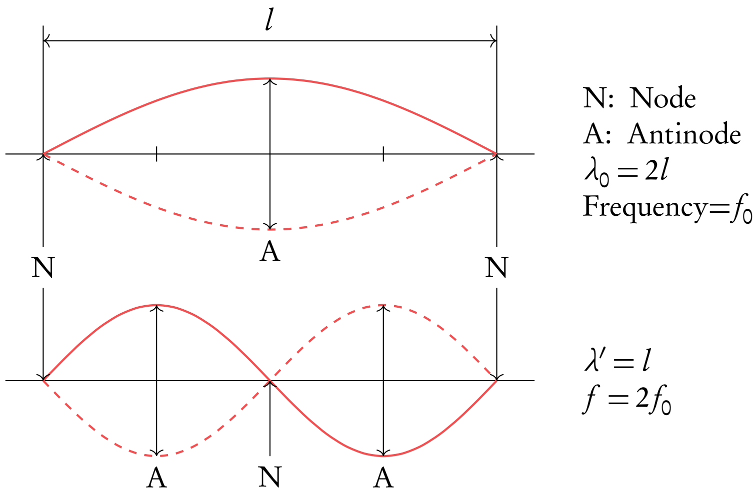
Imagine plucking the string of a guitar. What does the string do? It vibrates back and forth, at a certain frequency. Now if we put our finger right on the center of the string, this point will not be allowed to vibrate and the result is that the two ends will be vibrating back and forth inversely to one another. This is shown in the Figure 4.4.

Standing wave is a wave in a medium in which each point on the axis of the wave has an associated constant amplitude.

A node is a point on the standing wave that does not oscillate.

An antinode is a point on the standing wave that is always oscillating to the maximum amplitude.

Figure 4.4: Vibrating guitar string.



There are several notable differences between standing waves and travelling waves:

Standing

- Energy is stored
- Maximum amplitude depends on position
- All points between nodes have equal phase

Travelling

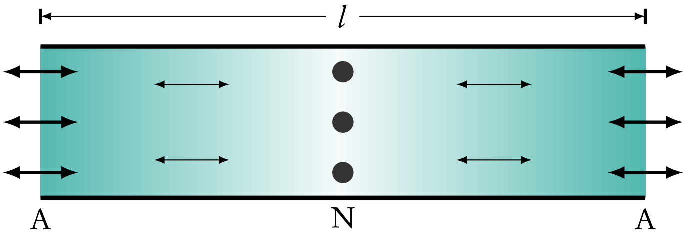
- Energy is transmitted
- Maximum amplitude is universal

When solving problems involving standing waves, we will focus on:

- The length of the wave, L
- Its wavelength, λ
- Its harmonic frequencies, f

It is important to know the various boundary conditions that may lead to standing waves. Above we already gave the example of a fixed string, however, in the IB you should also be aware of the various systems set up in pipes.

Figure 4.5: Open pipe.



Every standing wave has a fundamental frequency corresponding to the most simple oscillation, for a system open at both ends the fundamental frequency is calculated by way of fundamental wavelength:

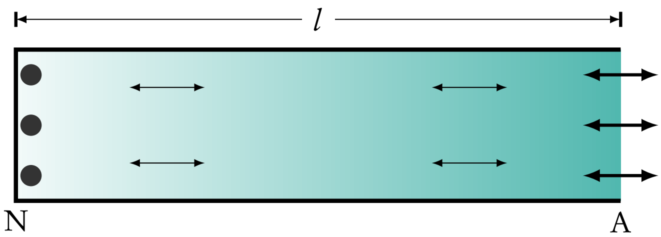
$$\lambda_0 = 2L$$

where λ_0 is the fundamental wavelength and L is the length of the standing wave. To calculate the rest of the harmonic wavelength or frequencies we use the relation:

$$2L = n\lambda$$

In both instances we may use $f = \frac{v}{\lambda}$ to calculate the corresponding frequency.

Figure 4.6: Half open pipe.



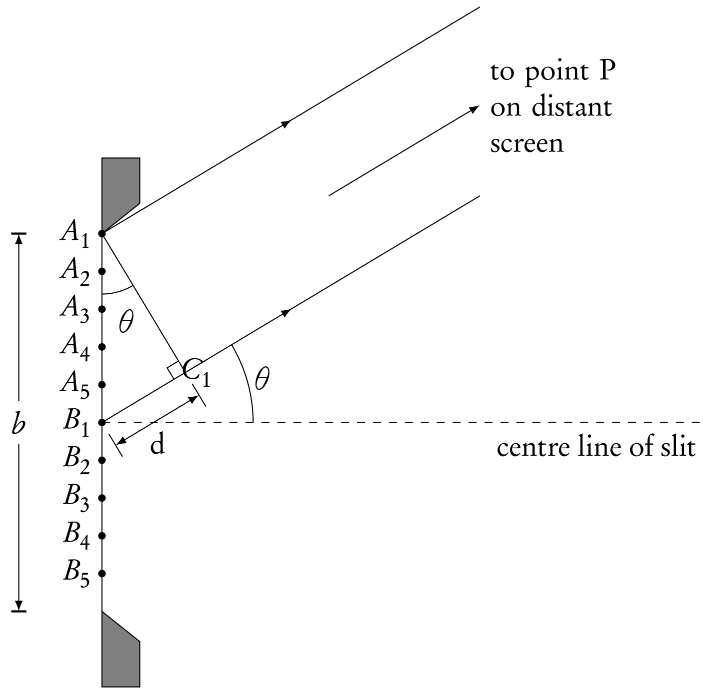
For a system closed at one end, the fundamental wavelength may be calculated using:

$$2L = \left(n - \frac{1}{2}\right)\lambda$$

This added factor of $\frac{1}{2}$ can be accounted for by the half-wavelength phase difference that occurs in a closed system, that is, there is a node at the closed end of the wave and an antinode at the open end. This equation may be used specifically to calculate the harmonic frequencies in a one-end open pipe.

Single-Slit Diffraction

Single-slit diffraction When a wave of wavelength λ is incident on an aperture of size b, diffraction takes place where the wave spreads out past the aperture.



The key assumption is that λ is of equal magnitude as b. When diffraction occurs, there is interference between the wavefront. The interference depends on the path difference
DB BC. Using trigonometric rules we can deduce:

$$\text{path difference} = BC = \frac{b}{2}\sin\theta$$

Assuming that $A_1B_1 = A_2B_2 = A_3B_3$, we have parallel identical wave-fronts and the path difference is valid for determining the interference pattern seen on the screen.

Minimum Condition

$$b \sin\theta = \lambda$$

as θ is small, then $\sin\theta \sim \theta$ so we have:

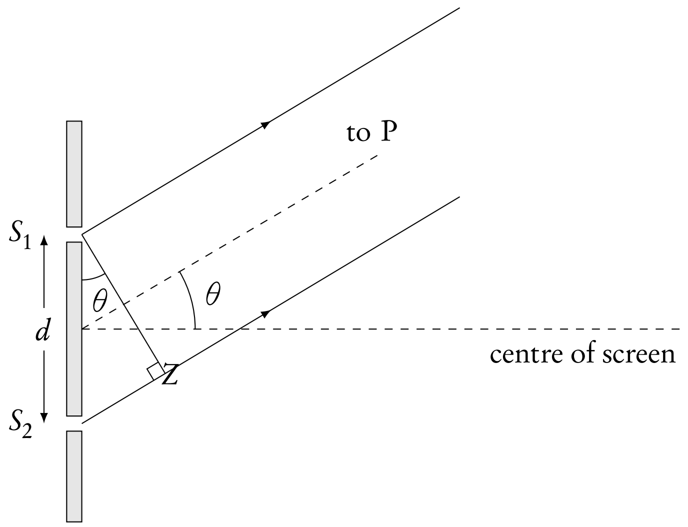
DB

$$\theta = \frac{\lambda}{b}$$

Interference the superposition of waves, increasing or decreasing the amplitude

Constructive interference path difference $= n\lambda$

Destructive interference path difference $= \left(n + \frac{1}{2}\right)\lambda$



The following conditions may be used to determine the type of interference.

	One phase change	No or two phase changes
Constructive	$2dn = \left(m + \frac{1}{2}\right)\lambda$	$2dn = m\lambda$
Destructive	$2dn = m\lambda$	$2dn = \left(m + \frac{1}{2}\right)\lambda$

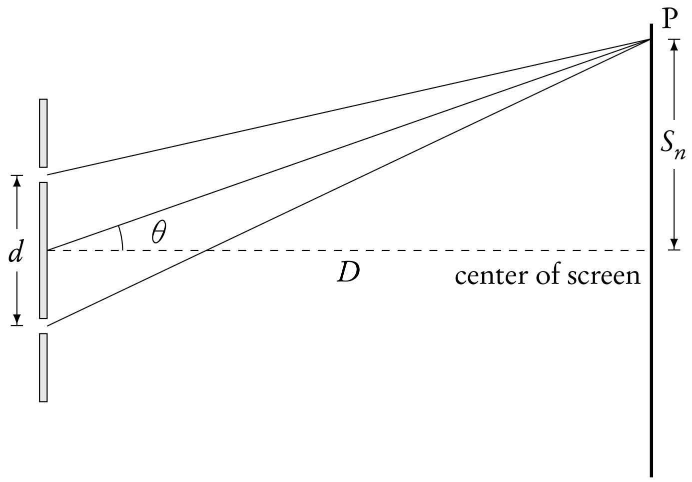
Two-slit setup

For constructive interference in two-slit setup:

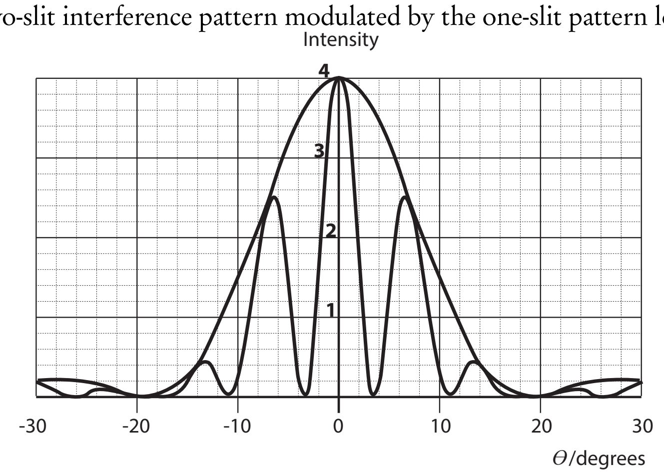
$$d \sin\theta = n\lambda$$

DB We may want to measure the distance S_n to the n^{th} fringe on the screen.

$$S_n = \frac{n\lambda D}{2}$$



An example of two-slit interference pattern modulated by the one-slit pattern looks like

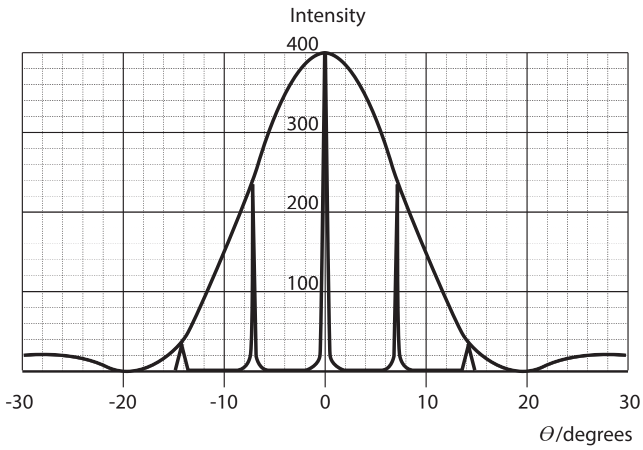


where $b = 3\lambda$ and $d = 8\lambda$.

What happens when we increase the number of slits as in a diffraction grating?

- primary maxima will become thinner & sharper
- $N-2$ secondary maxima becomes unimportant

- intensity of central maxima $\propto N^2$

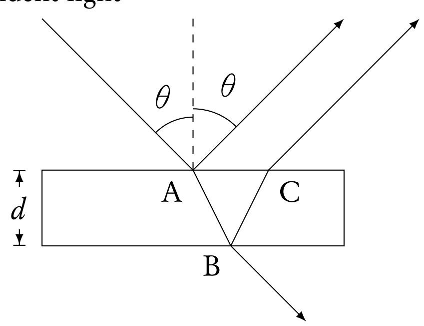


Diffraction grating consists of a large number of slits of negligible width. It is mainly used to measure the wavelength of light.

Diffraction gratings are usually defined in terms of 'x lines per mm', thus $d = \frac{1}{x}$ mm.

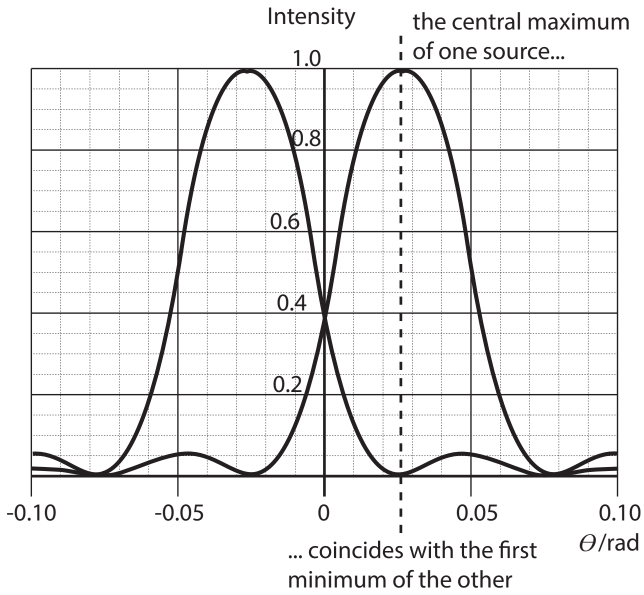
Thin-film Interference

The colours seen when oil has spilled on the street is due to thin film interference.



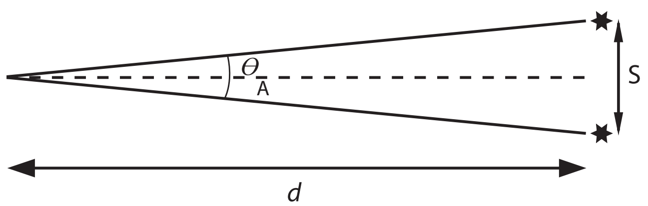
Resolution the distance/size of objects to observe them as distinct objects.

The Rayleigh criterion determines when two objects can just be resolved, and occurs when the first minimum of one pattern coincides with the central maximum of the other.



Solving resolution problems involves the comparison of the two angles.

$$\theta_A = \frac{S}{d} \qquad \text{(angular separation)}$$



$$\theta_D = \frac{\lambda}{b}(1.22) \qquad \text{(circular slit diffraction condition)}$$

Two objects are resolved if:

$$\theta_A \geq \theta_D$$

or

$$\frac{S}{d} \geq \frac{\lambda}{b}(1.22)$$

An important characteristic of a diffraction grating is its ability to resolve. DB

The resolving power R of a grating is

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = mN$$

λ_{avg} = average wavelength
$\Delta\lambda$ = difference between two lines
m = order of the lines
N = total number of slits

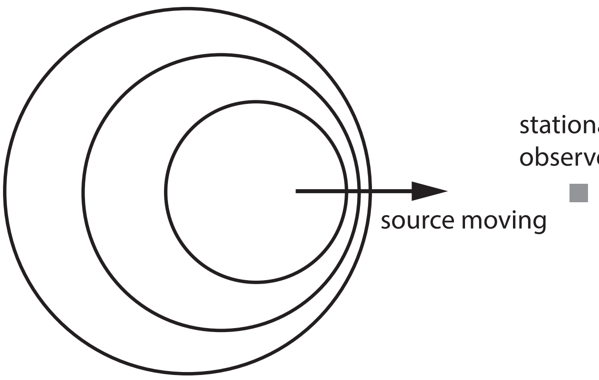
Doppler Effect

The Doppler effect is the change in observed frequency of a wave which happens whenever there is relative motion between the source and the observer.

Consider the following four situations:

1. A source approaching a stationary observer with speed u_s. The observed frequency is:

$$f' = f\left(\frac{v}{v - u_s}\right)$$



The stationary observer therefore measures a higher frequency.

2. A source moving away from a stationary observer. The observed frequency is:

$$f' = f\left(\frac{v}{v + u_s}\right)$$

The stationary observers measures a lower frequency.

3. An observer moving towards a source with velocity u_o (similar to the source approaching a stationary observer)

$$f' = f\left(\frac{v + u_0}{v}\right)$$

The observer measures a higher frequency.

4. An observer moving away from a source (similar to the source moving away from a stationary observer)

$$f' = f\left(\frac{v - u_0}{v}\right)$$

The observer measures a lower frequency.

Before choosing which formula to use, think about the situation intuitively to find out whether the measured frequency will be higher or lower.

ELECTRICITY AND MAGNETISM

5

5.1 Electric fields and potential

5.1.1 Charge

There are two types of charge, positive and negative, and they are the opposite of each other. Like charges repel each other while opposite charges attract.

Electric charge physical property of matter that causes it to experience a force to other electrically charged matter

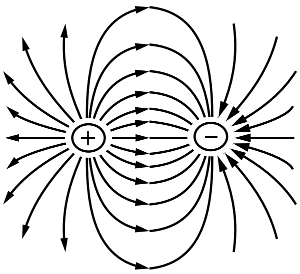
Conservation of charge just like energy, electric charges are conserved in all physical processes

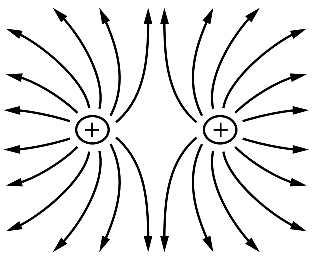
The SI derived unit of electric charge is the coulomb (C). Coulomb's law quantifies the force between two particles, which depends on the amount of charge and the distance between them.

Coulomb's law: force between charges

$$F = k_e \frac{q_1 q_2}{r^2}$$

F	force	[N]
k_e	Coulomb's constant	$8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$
q_1	charge	[C]
q_2	charge	[C]
r	distance	[m]





5.1.2 Nature of electric fields

Electric field   is the electrostatic force on a stationary test particle of unit charge 1 C

Note that test charges are always *positive*, unless the question states otherwise (e.g. an electron)

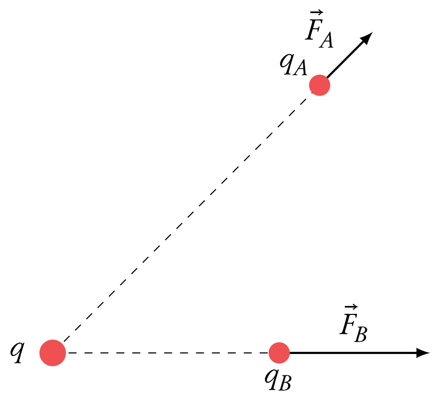
Electric fields are caused by electric charges, and the electric field strength diminishes further away from its source. Field strength can be visualised: the higher the field line density, the stronger the field.

Inside an electric field of a particular strength, the force exerted on a test charge depends *only* on the amount of charge.

The electric field strength is defined as the electrostatic force per unit charge.

$$E = \frac{F_E}{q}$$

E = electric field [NC^{-1}]
F_E = electrostatic force [N]
q = charge [C]



- particle q_A that is further away from q will experience a smaller force $\vec{F}_A$, than
- particle q_B that is closer to q which will experience a larger force $\vec{F}_B$

5.1.3 Potential difference

Since charge inside an electric field experience a force, moving a test particle requires work that changes its *electric potential energy*.

Voltage = electric potential difference ΔV the difference in electric potential energy of a charge that *moves* between two points *per* unit charge [$V = JC^{-1}$]

When some unknown amount of charge moves in an electric field, its potential difference changes. (much like when we lift an apple of unknown weight from the ground) The amount of electric potential difference depends on the strength of the electric field, and the distance the charge moved:

Electric potential difference by moving a charge in an electric field:

$$\Delta V = E \cdot \Delta x \quad (5.1)$$

V	=	electric potential difference	[V]
E	=	electric field	$[\mathrm{V\,m^{-1}}]$
x	=	distance	[m]

The electric potential difference ΔV is independent of the charge, since the value is *per* unit charge, so that once the charge is known the amount of energy (work) can be calculated.

Work done by moving a charge in an electric field:

$$W = \Delta V \cdot q \quad (5.2)$$

W	=	work	[J]
V	=	electric potential difference	[V]
q	=	charge	[C]

Since the amount of work done on an electron by a potential difference is very small, since the charge of the electron is very small, eV units are often used instead of J.

Electron volt eV: 1.602×10^{-19} J is the amount of work needed to change the electric potential of an electron by 1 V. It is a unit of energy!

To calculate the change in potential energy or work done by a charge in an electric field, substitute ΔV from (5.1) into (5.2), yielding the following equation:

Work done by a charge moving in an electric field:

$$W = E \cdot \Delta x \cdot q \quad (5.3)$$

W	=	work	[J]
E	=	electric field	$[\mathrm{V\,m^{-1} = N\,C^{-1}}]$
x	=	distance	[m]
q	=	charge	[C]

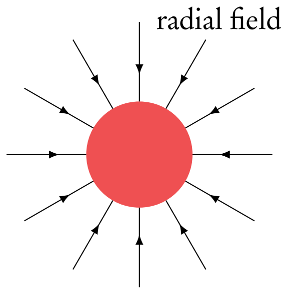
5.1.4 Applications

Electric field and electric potential inside a conducting sphere

The electric field inside a conducting sphere is *zero*.

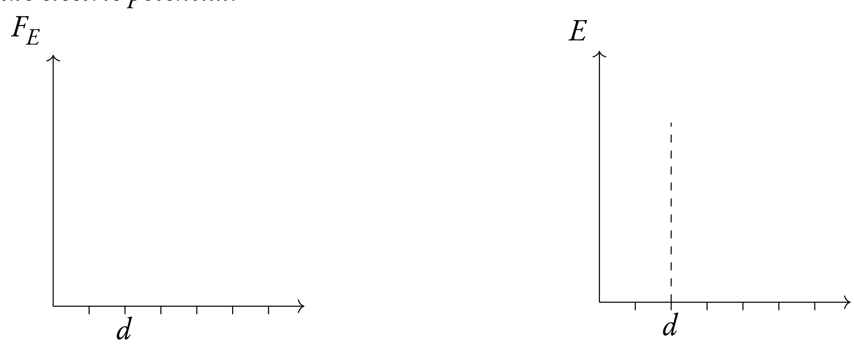
The electric potential inside a conducting sphere is *constant*.

Figure 5.1: A negatively charged conducting sphere



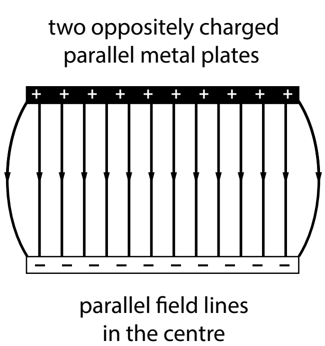
Exercize.

If the radius of a charged conducting sphere is d, draw the profile of the electric field and the electric potential.



Electric field between two parallel conducting plates

The parallel lines between the two plates means that the electric field is uniform.



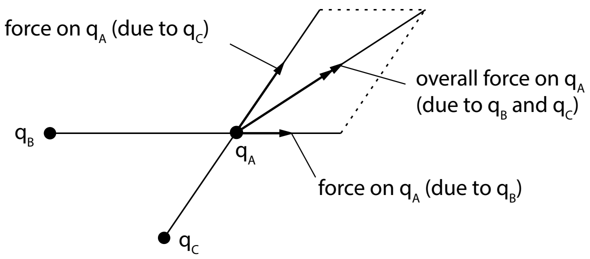
The field strength is calculated with a very simple equation:

$$E = \frac{V}{d}$$

E	= electric field	$[NC^{-1}]$
V	= potential difference	[N]
d	= separation	[C]

Electrostatic field due to multiple charges

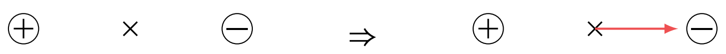
Lastly, you be might asked a question about the electrostatic field due to multiple charges. Looking at the superposition of electric field vectors easily solves this. Drawing a vector diagram makes it a lot easier. Make sure to pay attention to what charges are positive and negative, as this will influence the direction in which your test charge will want to go!

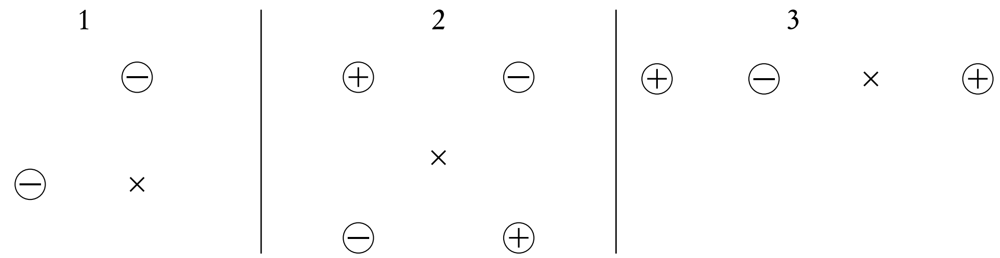


Vector addition of electrostatic forces

Exercize.

Draw the net field vector at the '×' for each of the situations. (Assume all charges are equal)

Example: 



5.2 Electric current

Electric current [A] the flow of electric charge, the amount of charge that flows per unit time

$$I = \frac{\Delta q}{\Delta t}$$

I = current [A]
q = charge [C]
t = time [s]

5.2.1 Direct current (dc)

While positive metal nuclei in metals are held in place and can only vibrate, electrons in a metal can move freely from one metal nuclei to another. The 'sea of electrons' are at all times homogeneously distributed throughout the metal, ensuring that the positive charge of the metal nuclei are at all times neutralised. When a potential difference across the metal is applied, electrons accelerate in the externally applied electric field proportional to the maginitude of that field. Because electrons bump into the a.o. metal nuclei, transferring some energy in the collision thus heating up the metal because the nuclei will vibrate more, the electrons attain a specific average velocity proportional to the electric field.

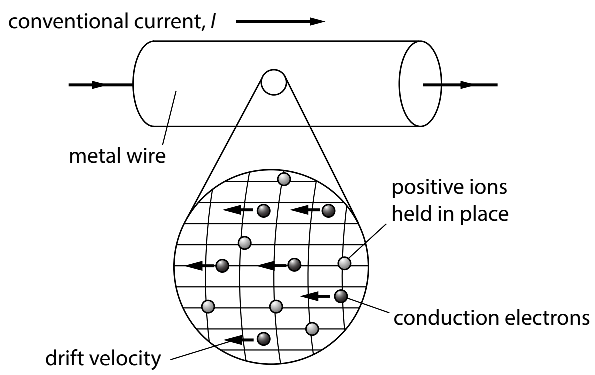
Drift velocity the average velocity of electrons in a material due to an electric field

More electrons per unit time will move [A]: 1. when a higher potential difference is applied [V], 2. when electrons can move through the material more easily and 3. the cross-section of the material is larger. 2 & 3 together [Ω]

Ohm's Law

$$R = \frac{V}{I}$$

R = resistance [Ω]
V = voltage [V]
I = current [A]



Resistance of a wire from material properties

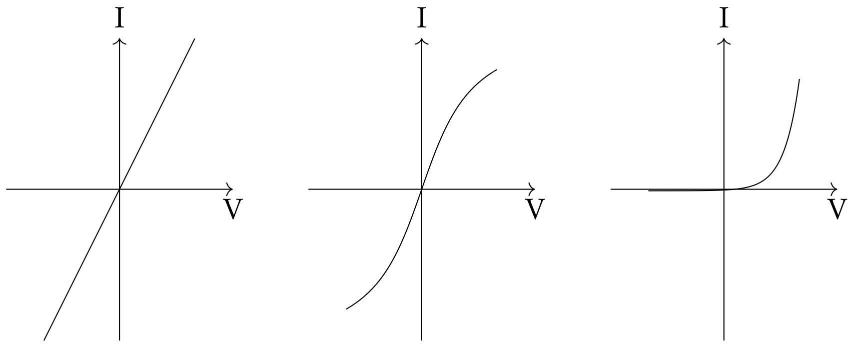
$$\rho = \frac{RA}{L}$$

which after rewriting gives

$$R = \rho\frac{L}{A}$$

ρ = resistivity of the material [$\Omega\,$m]
L = length of the wire [m]
A = cross-sectional area of the wire [m^2]

A device is ohmic if current and voltage are proportional, because resistance is *constant* at all values for current and voltage. When they are not proportional, the system is said to be non-ohmic.



(a) metal at constant T (ohmic)   (b) filament lamp (non-ohmic)   (c) diode (non-ohmic)

5.2.2 Power dissipation

Power dissipation resistance to electric current causes a material to warm up (when the electrons bounce off positive ions)

The power input/output of a system is the amount of energy per second that it uses/produces. Your laptop consumes a particular amount of energy per second [$J\,s^{-1}$], which is expressed in watts [W].

Power of electrical devices

$$\frac{\Delta E}{\Delta t} = P = VI$$

substituting Ohm's law into V or I gives

$$P = I^2R = \frac{V^2}{R}$$

P = power [W]=[$J\,s^{-1}$]
V = potential difference [V]
I = current [A]

While we often consider devices are power consumers, wires also consume power due to the resisivity of the material. To calculate the amount of electrical energy that is converted into heat by a wire, the above formula is often used in the form of $P = I^2R$. This formula also shows that powerloss in a wire is independent of the voltage!

5.3 Electric circuits

5.3.1 Resistors

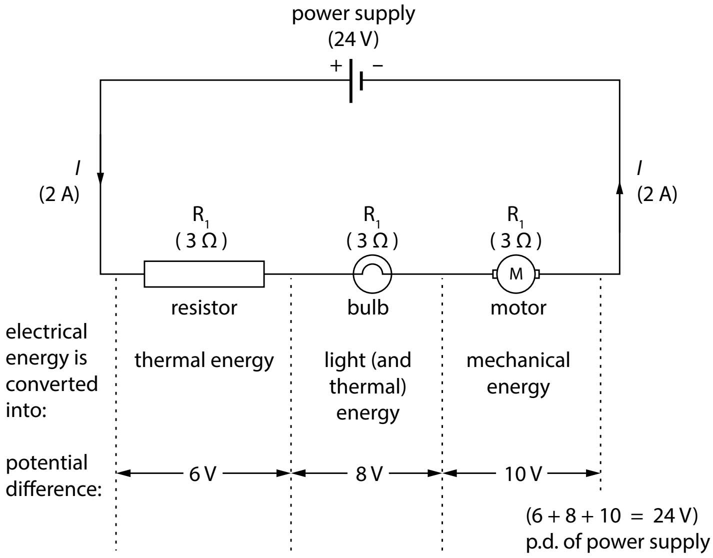
Resistors in series

- The potential difference is split between the resistors. That is, they may be summed linearly

$$V_{tot} = V_1 + V_2 + V_3$$
$$R_{tot} = R_1 + R_2 + R_3$$

- Current is constant throughout *all* resistors.

Figure 5.2



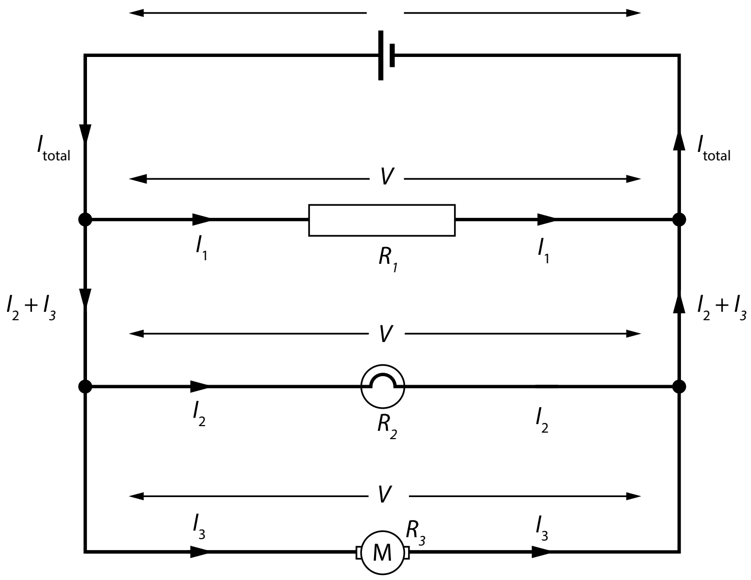
A variable resistor divides the potential difference accross it. The easiest way of looking at these is to consider two resistors in series: the potential difference is split between the resistors. When the resistance of these changes, so does the potential difference: so using the variable resistor we can vary the potential difference.

Resistors in parallel

- The potential difference is the same across all resistors in parallel.
- The current varies per resistor depending on the amount of resistance.

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$I_{tot} = I_1 + I_2$$

Figure 5.3



5.3.2 Electrical meters

Ammeter is a current-measuring meter. It needs to be connected in *series* at the point where the current is to be measured. An ideal ammeter would have zero resistance

Voltmeter is a meter that measures potential difference. It needs to be placed in *parallel* with the component or components being considered. An ideal voltmeter has infinite resistance.

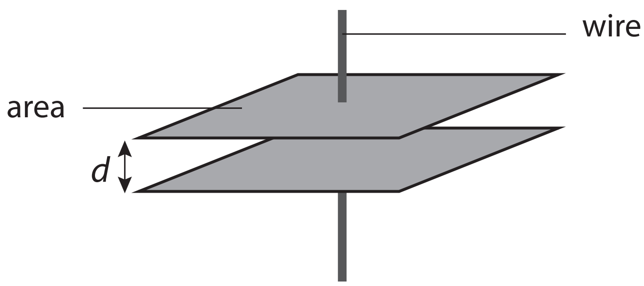
5.3.3 Capacitance

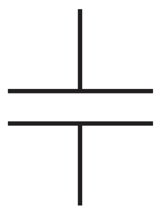
A capacitor is defined as an arrangement of two conductors separated by an insulating material.

Capacitance C is defined as the charge q per unit voltage V stored on the capacitor.

$$C = \frac{q}{V}$$

C	= capacitance	[F]
q	= charge	[C]
V	= voltage	[V]





circuit symbol for capacitor

For a parallel-plate capacitor we may write:

$$C = \varepsilon \frac{A}{d}$$

C = capacitance
ε = permittivity of the encapsulated medium
A = area of the one of the plates
d = separation of the plates

Insulators are also known as dielectric materials. Dielectric materials induce a larger capacitance as we can see from $C = \varepsilon \frac{A}{d}$ where ε, the dielectric constant is greater than ε_0, the dielectric constant of a vacuum. This happens because of charge polarisation which is basically just a separation of charge within the dielectric material.

Energy in a capacitor

The energy stored in a capacitor is given by:

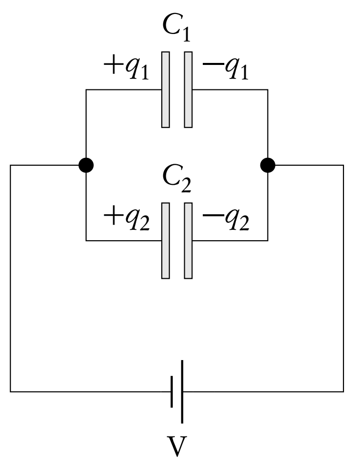
$$E = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

E = energy
C = capacitance
V = potential difference
q = final charge on capacitor

Example.

Parallel capacitors are cumulative

$$q_1 = C_1 V \quad \text{and} \quad q_2 = C_2 V$$

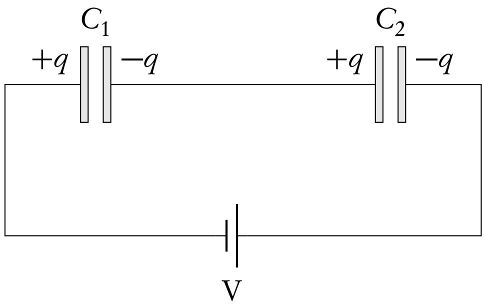


the total charge is

$$q = q_1 + q_2 = (C_1 + C_2)V$$
$$C_{\text{parallel}} = C_1 + C_2 + \ldots$$

Example.

Capacitors in series have the same charge as shown in the figure.



$$V = \frac{q}{C_1} + \frac{q}{C_2}$$
$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

5.4 Electromagnetic (EM) Induction

Key idea: this section deals with Faraday's law and how a magnetic flux through a loop induces an emf in the loop.

5.4.1 Electromotive force

Electromotive force (e.m.f.) is the largest potential difference that may be experienced by charges moving around the circuit.

Note that the e.m.f. or $\mathcal{E}$ is equivalent to V_{tot} or the potential difference across the circuit.

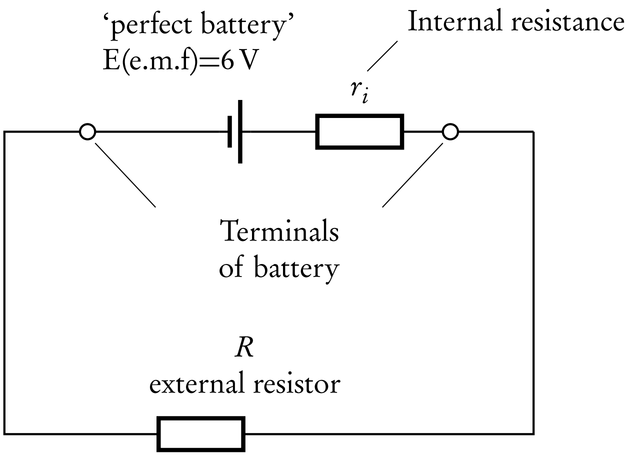
The battery that is providing the driving force of the circuit often has some form of internal resistance r. It is calculated by using the current I and the electromotive force $\mathcal{E}$ of the circuit, calculated using the following formula.

Electromotive force

$$\mathcal{E} = I(R + r)$$

$\mathcal{E}$ = electromotive force	[V]
I = current	[A]
R = external resistance	[Ω]
r = internal resistance	[Ω]

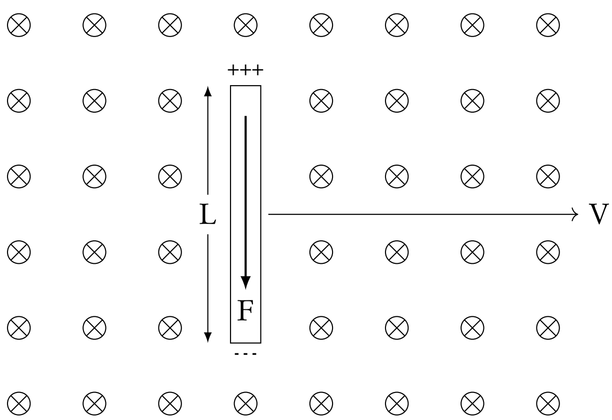
Figure 5.4



$$\begin{aligned} \text{e.m.f.} &= I \times R_{total} \\ &= I(r_i + R) \\ &= I r_i + IR \\ \underbrace{IR}_{\text{terminal p.d.}, V} &= \text{e.m.f.} - \underbrace{I r_i}_{\text{'lost' volts}} \end{aligned}$$

EMF from motion

Imagine a rod of length L that is moved with velocity v in a region of constant magnitude B.



This movement establishes an effective electric field due to electrons being pushed to one side of the rod. This field is given by

$$E = \frac{\mathcal{E}}{L}$$

where $\mathcal{E}$ is the induced emf.

The flow of electrons will stop when:

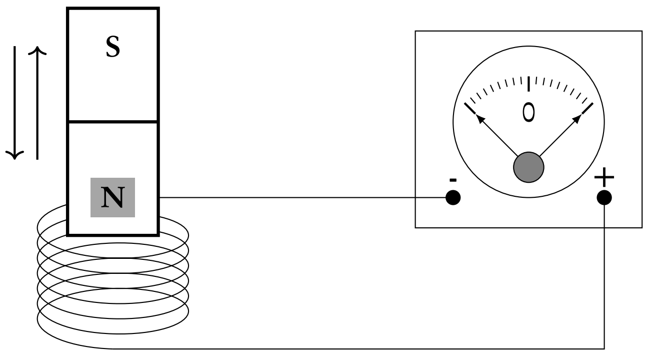
$$eE = evB$$

Thus the motional emf is:

$$\mathcal{E} = BvL$$

5.4.2 Magnetic flux & Flux linkage

When a magnet moves relative to a coil, a current is created as a result of the motion.



The current increases when:

- the relative speed increases
- the strength of the magnet increases
- the number of turns increases
- the area of the loop increases
- the magnet moves at right angles to the loop (steeper).

The common thread between these observations is magnetic flux.

The flux linkage through the loop is:

$$\phi = NBA\cos\theta$$

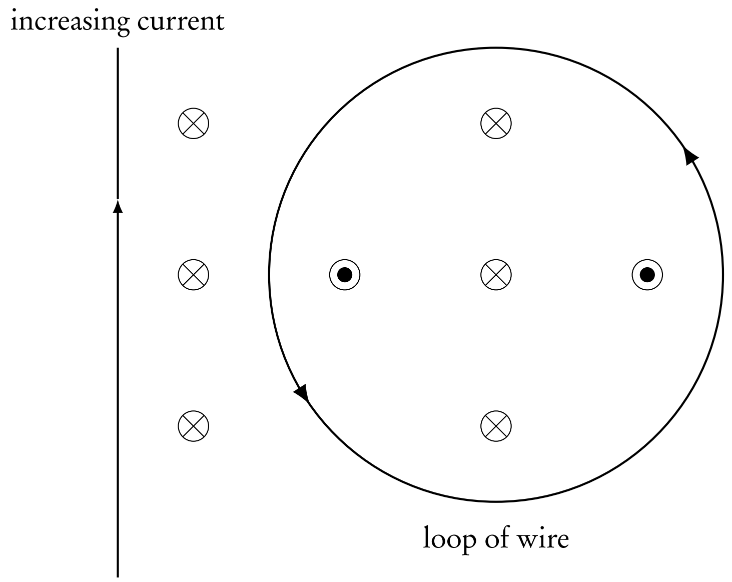
N = number of turns
B = magnetic field strength
A = area of the loop

Relation to current

Faraday's law states that the emf is proportional to the number of turns on the coil N, and the flux change $\Delta\phi$ over time Δt:

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t}$$

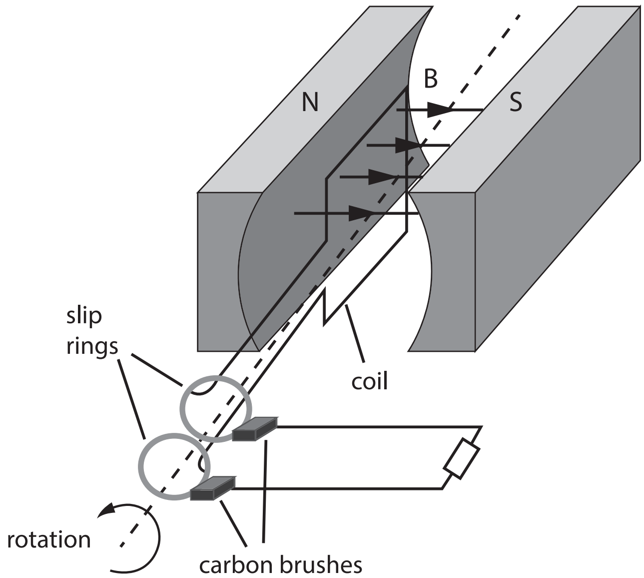
To find the direction of the induced current we have to consider Lenz's law. It states that the induced emf will oppose the change in magnetic flux that created the current.



5.4.3 Applications

Transmission of power

AC generators are used universally to generate electricity, in modern cars, power plants etc. Kinetic energy is converted to electricity using a generator:

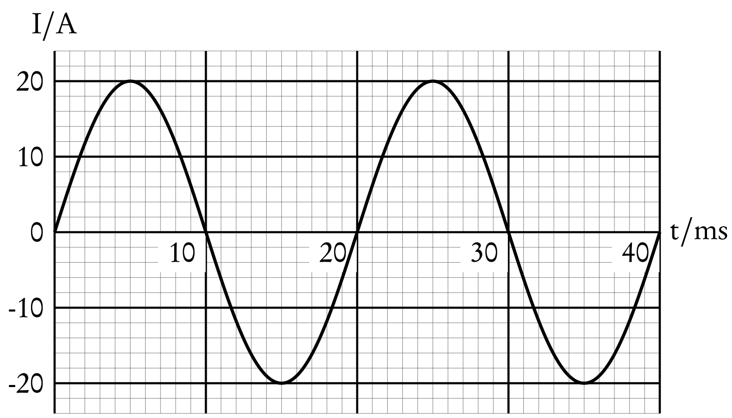


Using Faraday's law, the emf induced in the coil is:

$$V = -\frac{d\phi}{dt} = \omega NBA\sin(\omega t)$$

Current is found using $I = \frac{V}{R}$

$$I = I_0 \sin(\omega t)$$



Similarly, power is found using $P = VI$

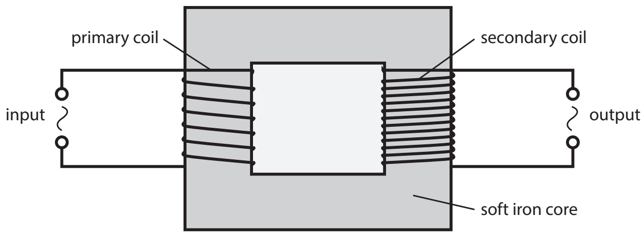
$$P = V_0 I_0 \sin^2(\omega t)$$

Often, we are more interested in root mean square (rms) quantities. The root mean square is used to find the average voltage, current and power. To calculate use:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$
$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$
$$P_{\text{avg}} = \frac{V_0 I_0}{\sqrt{2}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^2}{R}$$

RMS quantities are the average values of AC circuits, so those equal the values that DC circuits would have.

Transformer The transformer is a device that takes a certain AC voltage as input and delivers different AC voltage as output.



It does this by changing the number of turns from primary to secondary coil. The change in emf is given by

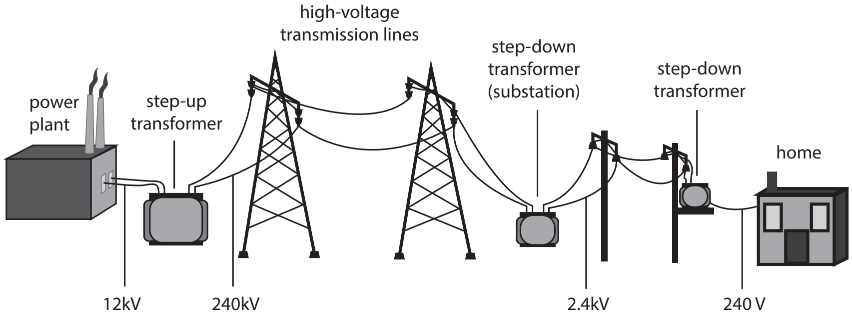
$$\frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s} \quad \Rightarrow \quad \mathcal{E}_p I_p = \mathcal{E}_s I_s \qquad \begin{cases} p = \text{primary} \\ s = \text{secondary} \end{cases}$$

Note: transformers are often used to step up the voltage to reduce power loss when transmitting electrical energy. The transformer changes the current and the voltage, but it does not change the frequency. Transformers undergo power losses from several sources.

Losses

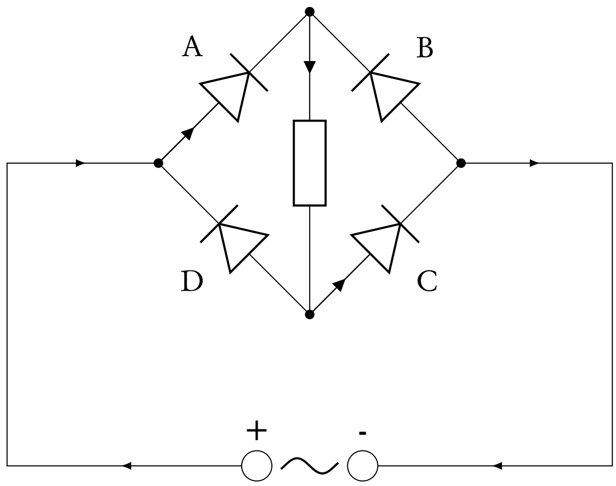
- Eddy currents are tiny currents created in the core because free electrons move in the presence of magnetic field.

- Magnetic hysteresis is a second source of loss in magnetic energy.



Rectification

To convert an AC current into a DC current, one can use a diode bridge rectifier to achieve full-wave rectification.



It's important to be able to draw this kind of diagram for your exam.

5.5 Magnetic fields

	E – electric	B – magnetic
Caused & affected by:	charges	magnets (or currents)
Two types:	positive and negative charge	north and south pole
Force rule:	like charges repel like charges attract	like poles repel like poles attract

Magnetic fields may be induced by the movement of charge.

Figure 5.5

Figure 5.6

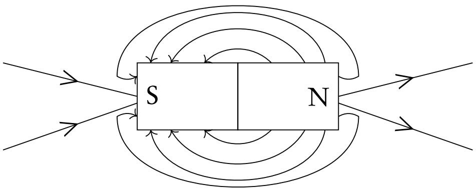
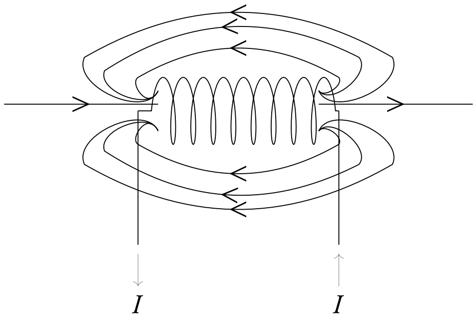


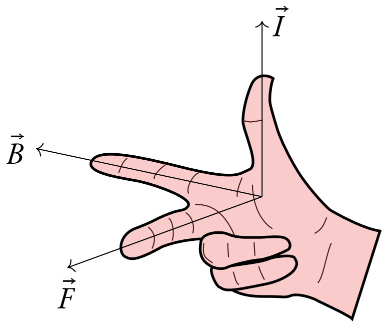
Figure 5.7



Right-hand rule

Middle $= F$ (force)
Pointer $= B$ (magnetic field)
Thumb $= I$ (current)

FBI

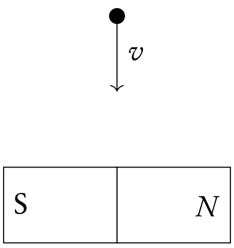


This rule is used whenever you are solving problems using either:

$$F = qvB\sin\theta \qquad \text{(moving charge)}$$
$$F = BIL\sin\theta \qquad \text{(current in a wire)}$$

Exercize.

An electron approaches a bar magnet as shown in the figure. If $v = 5.000\,\mathrm{m/s}$, $B = 5\,\mathrm{T}$.



1. Draw the field lines in the diagram.
2. Draw the direction of the force on the electron.
3. What is the magnitude of this force?

Using the equal and opposite forces of two current carrying wires on each other, we can define the ampere.

The ampere is defined through the magnetic force between two parallel wires. If the force on a 1 m length of two wires that are 1 m apart and carrying equal currents is 2×10^{-7} N, then the current in each wire is defined to be 1 A.

CIRCULAR MOTION AND GRAVITATION

6

ACADEMY

6.1 Circular motion

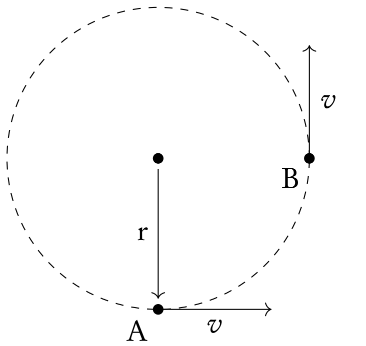
Circular motion is common in everyday life. An interesting example is the motion of planets around the Sun in nearly circular orbits. The pre-requisite for circular motion is a force directed towards the centre, a centripetal force.

Centripetal acceleration A body moving along a circle of radius R with speed v experiences centripetal acceleration given by

$$a = \frac{v^2}{r}$$

and is directed towards the centre.

Example.



a) Draw the force vector at point A & B.

b) Draw the direction of the net force as the object moves from point A → B,

Exercize.

1. A car is on a racing track and wants to take a hair-pin needle corner as fast as possible. The driver happens to know that the tires can exert a maximum friction force of 20.000 kN.
2. Assuming a hemispherical hairpin corner with $r = 20\,\text{m}$, what is the maximum velocity with which a 500 kg race car can take the corner?
3. There's an even tighter corner right after the last one with $r = 15\,\text{m}$. If they want to maintain the same velocity how much mass should be removed from the car?

6.2 Gravitational field

We have briefly discussed gravity in mechanics, in the context of the acceleration we feel on Earth. However, as Newton famously pointed out in his glory days, gravity may be generalized to encompass the entire universe. This is why it has been coined Newton's universal law of gravitation. Although Einstein accounted for high-energy anomalies, this law still holds strongly when discussing non-relativistic scenarios.

6.2.1 Newton's universal law of gravitation

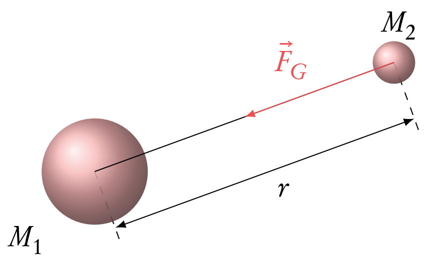
Newton's universal law of gravitation:

DB

$$F_G = G\frac{M_1M_2}{r^2}$$

F_G = gravitational force [N]
M_1 = first mass [kg]
M_2 = second mass [kg]
r = distance [m]
G = gravitational constant $6.674 \times 10^{-11}\,\mathrm{N\,m^{-2}\,kg^{-2}}$

- Gravitational force is always attractive.
- It is only *significant* for very massive objects (i.e. planets and stars).
- Spheres can be treated as point particles because the center of mass is located at the center.



6.2.2 Gravitational field strength

The gravitational field strength, g, is the value we use to define acceleration due to gravity.

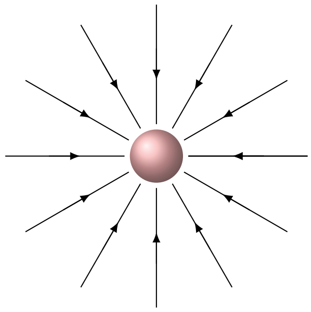
The gravitational field is defined as the force due to gravity per unit mass.

$$g = \frac{F_G}{m} = G\frac{M}{r^2}$$

g = acceleration $[\mathrm{m\,s^{-2}}]$
G = gravitational constant $6.674 \times 10^{-11}\,\mathrm{N\,m^{-2}\,kg^{-2}}$
M = mass [kg]
r = distance [m]

In mechanics, we use the correct value for Earth, $\approx 9.81\,\mathrm{m\,s^{-2}}$. However, the gravitational field strength can be easily derived from the law of gravitation.

Field diagrams are used when describing all three kinds of fields mentioned earlier (electric, magnetic & gravity), for gravitational fields these are the most straightforward as gravity will only attract other massive objects. The field will always point the way that any massive object moves.

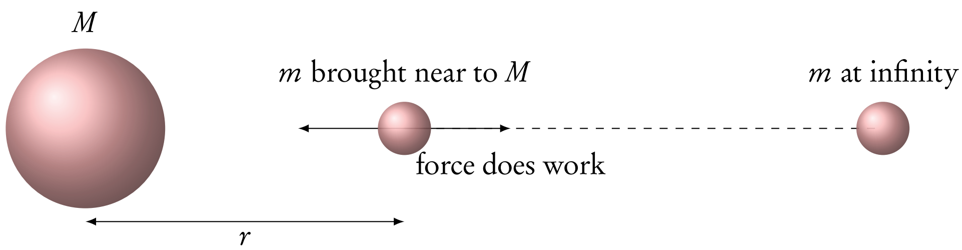


Exercize.

Now try drawing the field lines for these massive objects:

6.2.3 Gravitational potential energy of two bodies

The gravitational potential energy of two bodies is the work that was done in bringing the bodies to their present position from where they were infinitely apart.

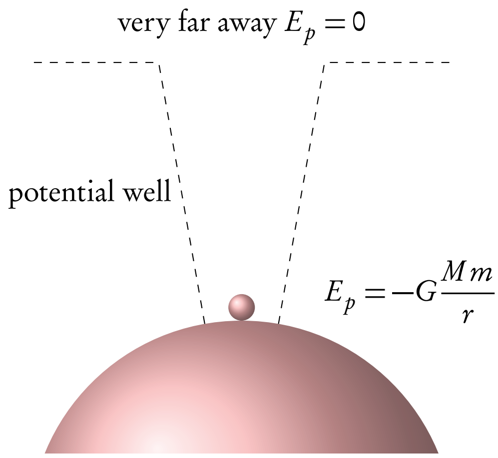


$$W = E_p = -G\frac{Mm}{r}$$

W	= gravitational potential energy	[J]
G	= gravitational constant	
M m	= masses	[kg]
r	= distance	[m]

ACADEMY

This energy is negative meaning we would need to supply $E = G\frac{Mm}{r}$ to separate the masses to infinity. It can be intuitively grasped as a potential well as shown.



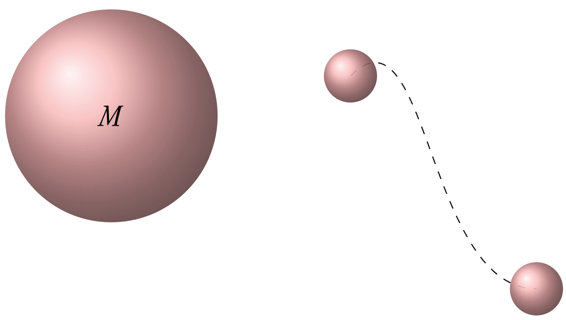
Gravitational potential at a point P in a gravitational field is the work done per unit mass in bringing a small point mass m from infinity to point P.

$$V_g = \frac{W}{m} = -G\frac{M}{r}$$

Remember this definition like its the back of your hand!

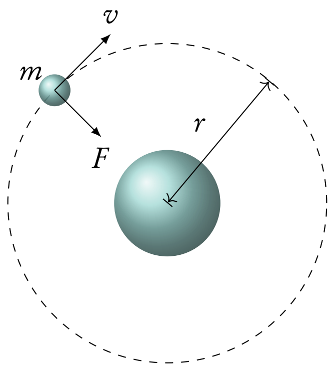
We can find the work done by moving in gravitational potential by regarding the mass and charge in potential.

$$W = m(V_gB - V_gA)$$



Example.

Fields at work. Orbital motion deriving orbital speed.



In orbital motion we can assume the presence of two forces.

$$F = m\frac{v^2}{r}$$
$$F = G\frac{Mm}{r^2}$$

To find the orbital speed we equate the forces

$$F = m\frac{v^2}{r} = G\frac{Mm}{r^2}$$
$$v^2 = G\frac{M}{r}$$
$$v = \pm\sqrt{G\frac{M}{r}} \qquad \text{(orbital speed)}$$

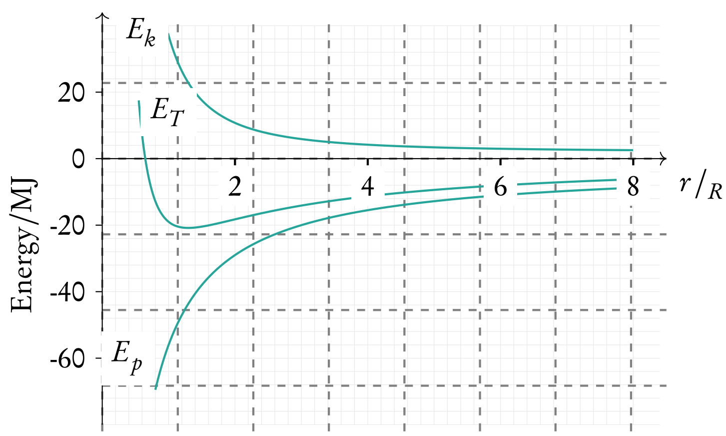
If we take a step back we can also find orbital kinetic energy:

$$m\frac{v^2}{r} = G\frac{Mm}{r^2}$$
$$mv = G\frac{Mm}{r}$$
$$E_k = \frac{mv^2}{2} = G\frac{Mm}{2r}$$

We can combine this with the gravitational potential energy discussed in the previous section to find the total orbital energy E_T.

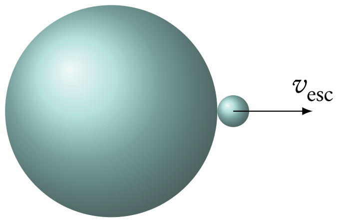
$$E_T = G\frac{Mm}{2r} - G\frac{Mm}{r} = -G\frac{Mm}{2r}$$

This is shown graphically below.



Example.

Escape velocity



Suppose a projectile of mass m is launched from a planet of mass M and radius R, with speed v.

$$E_T = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

At infinite distance $E_p = 0$ and $E_k = 0$ thus $E_T = 0$. Thus by conservation of energy:

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0$$

$$v^2 = \frac{2GM}{r}$$

$$v_{esc} = \pm\sqrt{\frac{2Gm}{r}} \qquad \text{(escape velocity)}$$

6.2.4 Similarities between gravitational and electrostatic fields

Electrostatic fields behave nearly analogous to gravitational fields.

Newton's law	Coulomb's law
$F_G = G\frac{M_1M_2}{r^2}$	$F_E = k\frac{q_1q_2}{r^2}$

- We assume the use of point charges.
- Forces act along the connection between the two points.
- Both gravitational and electrostatic forces are additive.
- Both fields are proportional to the inverse of the square of the distance.

The main difference that must be taken into account is that electric charge may be either positive or negative. It follows that opposite charges will attract but like charges will repel, while the gravitational force is always attractive.

ATOMIC AND NUCLEAR PHYSICS

7

ACADEMY

7.1 Atomic structure

Atoms differ in how many protons, neutrons, and electrons they have. These apparently small differences do make it impossible for us to have such a wide variety of materials.

7.1.1 Atomic models

Plum pudding model protons, neutrons and electrons are homogeneously distributed in the atom

Rutherford model protons and neutrons reside in a small and dense nucleus, electrons are homogeneously distributed

Bohr model protons and neutrons reside in a small and dense nucleus, electrons reside in atomic orbitals around the nucleus (electron cloud)

Subatomic particles

	Mass	Relative mass	Charge	Relative charge	Notation
Proton	1.67×10^{-24} g	$1\,\mathrm{g\,mol^{-1}}$	1.60×10^{-19} C	+1	p^+
Neutron	1.67×10^{-24} g	$1\,\mathrm{g\,mol^{-1}}$	0 C	0	n
Electron	9.11×10^{-28} g	$\approx 0\,\mathrm{g\,mol^{-1}}$	-1.60×10^{-19} C	−1	e^-

Proof of a small, dense and positively charged nucleus

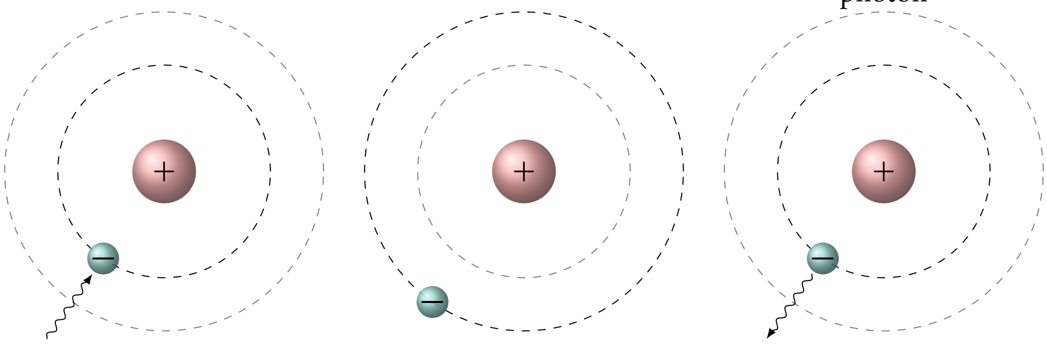
In the Rutherford experiment a beam of α particles was directed at a thin metal foil and the scattering pattern was measured by using a fluorescent screen. Thomson's model of the atom incorrectly predicted the alpha particles to go straight through the foil, instead a fraction of the particles were scattered (forced to deviate from a straight trajectory) by:

- Electrostatic repelling force between positive α particle and positive metal nuclei proved that the positive charge was concentrated
- A small number of collisions with metal nuclei caused a deflection of over 90°, proving that the metal nuclei were dense and confined to a region of linear size approximately equal to 10^{-15} m.

7.1.2 Emission and absorption spectra

Electrons in atoms can have very particular (discrete) amounts of energy, this means that the energy levels of electrons are quantized. Electrons can absorb and emit energy in various ways, one is way is in the form of absorbing and emitting photons.

Electron in ground state absorbs a photon	Energized electron jumps to a wider orbit	Electron returns to its ground state, emitting a photon



Discrete light quantum the amount of photon energy absorbed/emitted, equal to the energy difference of two electron energy levels

$$E = hf = \frac{hc}{\lambda}$$

E = energy
h = Planck's constant
f = frequency
c = speed of light
λ = wavelength

Emission

Emission an electron can emit a photon with a particular amount of energy, the electron will be in a lower 'excited energy state' or in the 'groundstate'

Atoms with electrons in the excited energy state emit light by 'falling down' to lower energy levels. The energy of the photon emitted equals the energy difference of two electron energy levels. When such emission light is passed through a prism emission lines are observed, each line represents the energy difference between two electron energy levels.

Figure 7.1: Emission spectrography setup.

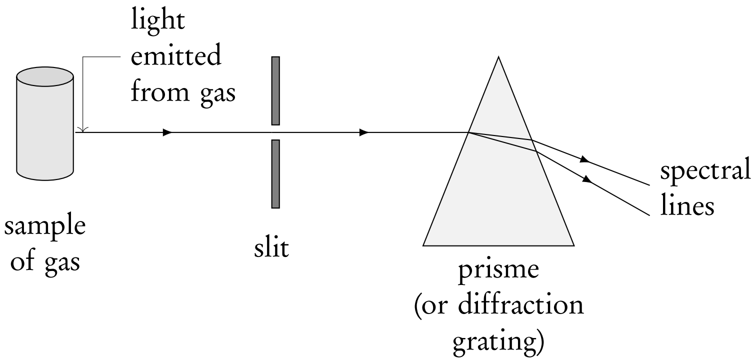
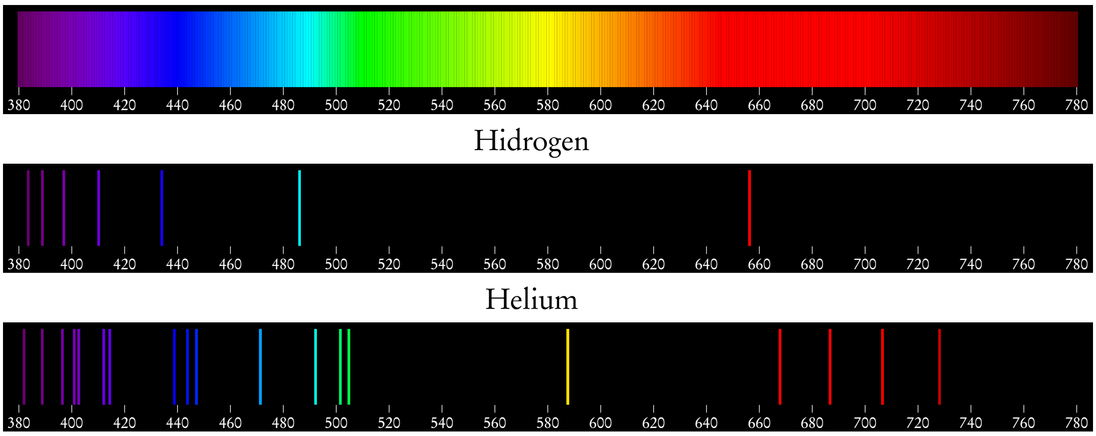


Figure 7.2: Emission spectrum

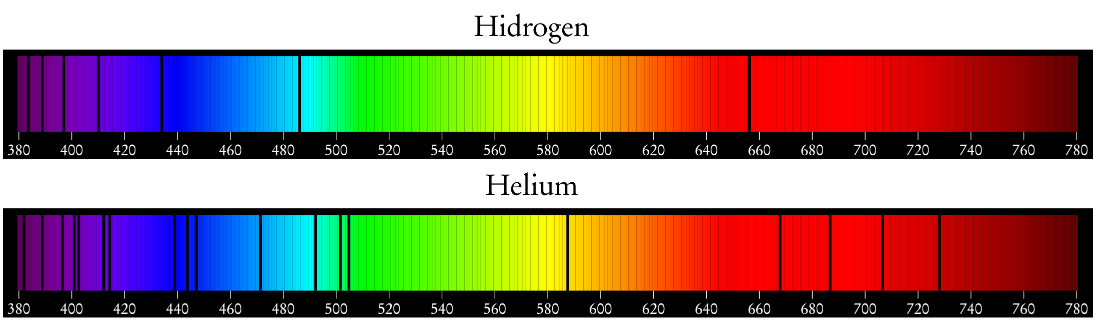


Absorption

Absorption an electron can absorb a photon with a particular amount of energy, the electron will be in a higher 'excited energy state'

When white light is passed through a sample with gaseous atoms, photons with energy equal to the energy difference of two electron levels can be absorbed. The remaining light is then passed through a prism, resulting in a spectrum wherein light of particular wavelength is absent.

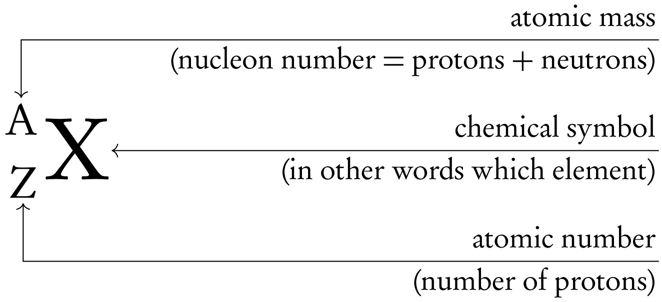
Figure 7.3: Absorption spectrum



7.2 Nuclear structure

Nucleon is either a neutron or a proton.

Nuclide is a nucleus that contains a specified number of protons and a specified number of neutrons.

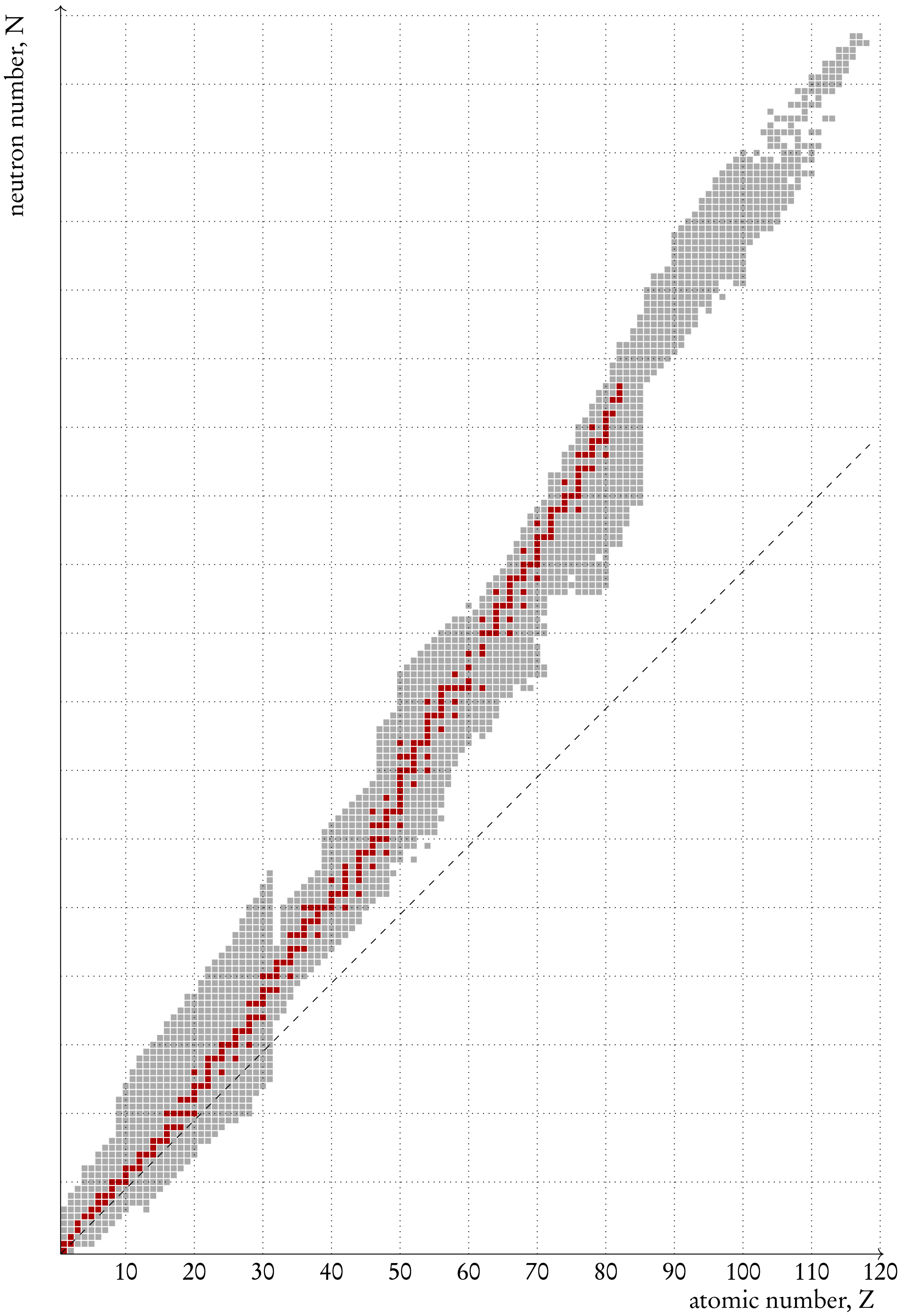


Isotopes a species of an atom that contains a specified number of protons and a specified number of neutrons (like a nuclide), and electrons. While the term isotope refers to the entire atom (including electrons), nuclide refers to only the nucleus.

Since all protons are in the nucleus, and like charges repel, there must be a force that holds the protons together and prevents them from falling apart. This force is called the strong nuclear force, and there are a few phenomena that we know this must adhere to:

- It must be strong, as the gravitational attraction is not nearly enough to overcome the electrostatic force that is repelling the protons from each other.
- As the force cannot be observed anywhere other than within the nucleus, it must act on a very short range
- It is likely to involve neutrons as well. Small nuclei tend to have equal number of protons and neutrons. Large nuclei need proportionally more neutrons in order to remain stable.

The graph below shows the relationship between the atomic number and the number of neutrons in a range of elements.



7.3 Radioactivity

Radioactive decay when an unstable nucleus emits a particle (alpha α, beta β, gamma γ).

- Note that radioactive decay is both a random and spontaneous process.
- Note that the rate of radioactive decay decreases exponentially with time.
- Note that radiation that originates from the atom is often the result of electron transitions. These do not fall under the definition of radioactivity.

Alpha decay when a chunk of the nucleus of an unstable atom is emitted in the form of a helium nucleus ${}^{4}_{2}He^{2+}$. What is left of the atom will have to be such that the protons and neutrons balance on each side of the equation.

$$ {}^{A}_{Z}X \longrightarrow {}^{(A-4)}_{(Z-2)}Y + {}^{4}_{2}\alpha $$

Beta decay, ${}^{0}_{-1}\beta$ or ${}^{0}_{-1}e^{-}$ occurs when a neutron in an unstable atom breaks up into a proton and an electron, of which the electron is then emitted along with an antineutrino.

$$ {}^{1}_{0}n \longrightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta + \overline{\upsilon} $$

such that

$$ {}^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}A + {}^{0}_{-1}\beta + \overline{\upsilon} $$

Gamma decay ${}^{0}_{0}\gamma$ is somewhat different from the other types of radiation in that they are part of the electromagnetic spectrum. After their emission, the nucleus has less energy but its mass number and its atomic number have not changed. It is said to have changed from an exited state to a lower energy state.

$$ {}^{A}_{Z}X^{*} \longrightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma $$

Effects of radiation

All three types of radiation are *ionizing*. This means that as they go through a substance, collisions occur which cause electrons to be removed from atoms, forming ions. When ionization occurs in biologically important molecules, such a DNA, it could cause cells to stop working or multiplying, or even become malignant or harm other cells. If this process perseveres, the group of malignant cells together forms what we call cancer.

Property	Alpha (α)	Beta (β)	Gamma (γ)
Effect on photographic film	Yes	Yes	Yes
Penetration ability	Low	Medium	High
Typical material needed to absorb it	10^{-2} mm aluminum; paper	A few mm aluminum	10 cm lead
Approximate number of ion pairs produced in air	10^4 per mm travelled	10^2 per mm travelled	1 per mm traveled
Typical path length in air	A few cm	Less than one m	Effectively infinite
Deflection by E and B fields	Behaves like a positive charge	Behaves like a negative charge	Not deflected
Speed	$\approx 10^7\ \mathrm{m\,s^{-1}}$	$\approx 10^8\ \mathrm{m\,s^{-1}}$, highly variable	$3 \times 10^8\ \mathrm{m\,s^{-1}}$ (speed of light)

Half-life

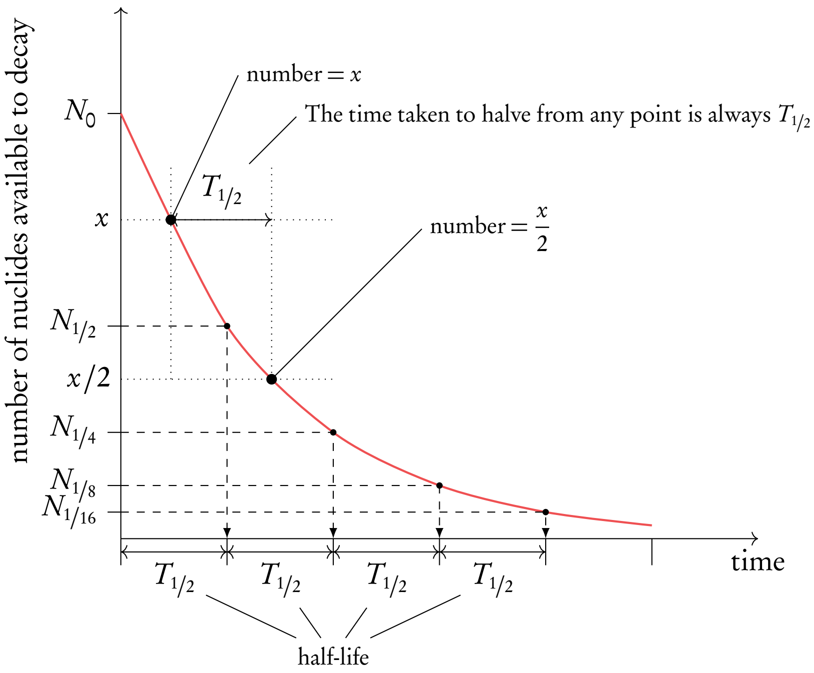
Radioactive decay of a single specific nucleus is not predictable since it is a random process, we can only know the chance of a decay occurring within a period of time. The decay rate of a sample is proportional to the number of atoms in the sample, which means that radioactive decay is an exponential process.

The number of atoms of a certain element, N, decreases exponentially over time. Mathematically this is expressed as:

$$\frac{\mathrm{d}N}{\mathrm{d}t} \propto -N$$

In the graph shown below, the time taken for half the number of nuclides to decay is always the same, whatever starting value we choose. This allows us to express the chances of decay happening in a property called half-life, $T_{1/2}$.

Radioactive half-life the time taken for half the number of radioactive nuclei in a sample to decay. Half-lives can vary from fractions of a second to millions of years.



Note that if the half-life of a nuclide is 3 days, the whole sample will not be decayed in 6 days, but rather half a half will remain, that is a quarter.

Radioactivity follows an exponential function. Using the method shown below, the decay constant (λ) can be found which can be used to find any relationship between the time and radioactive nucleons present in a sample.

Example

1.	$N = N_0 e^{-\lambda t}$	so $\frac{N}{N_0} = e^{-\lambda t}$
2.	If $\frac{N}{N_0} = \frac{1}{2}$	Then $\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$ where $t_{1/2}$ is the half life
3.	$\ln(2)^{-1} = -\ln(2) = -\lambda t_{1/2}$	$\lambda = \frac{\ln(2)}{t_{1/2}}$ $\rightarrow$ the decay constant

7.4 Nuclear reactions

Artificial (induced) transmutation when a nucleus is bombarded with a nucleon, an alpha particle or another small nucleus, a nuclear reaction is initiated resulting in a nuclide with a different proton number (a different element)

First, the target nucleus 'captures' the incoming particle and then an emission takes place.

$$^{4}_{2}\mathrm{He}_2 + ^{14}_{7}\mathrm{N} \longrightarrow ^{17}_{8}\mathrm{O} + ^{1}_{1}\mathrm{p}$$

The individual masses involved in nuclear reactions are tiny ($\sim 10^{-27}$ kg), so in order to compare atomic masses, physicists often use unified mass units, u.

Unified Atomic Mass Unit $\frac{1}{12}$ the mass of Carbon-12, approximately equal to the mass of 1 proton / 1 neutron

Mass defect

Mass defect is the difference between the total mass of the nucleus and the sum of the masses of its individual nucleons

Bringing protons together in a nucleus requires work to overcome the electrostatic repelling force, while together with neutrons strong nuclear forces form that release energy. The strong nuclear forces that bond protons and neutrons is larger than the electrostatic repelling force, so there is a net release of energy. Conservation laws state that this energy must come from somewhere: Einstein's famous mass-energy equivalence relationship.

$$E = mc^2$$

E	= energy	[J]
m	= mass	[kg]
c	= speed of light	[m s^{-1}]

1 kg of mass is equivalent to 9×10^{16} J of energy, which is a huge amount of energy. At the atomic scale the electronvolt (eV) is typically used instead of joule (J).

$$1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$$
$$1\,\text{MeV} = 1.6 \times 10^{-13}\,\text{J}$$

1 u of mass converts into 931.5 MeV

Mass defect calculations are very delicate given the long decimals, don't be intimidated by this and simply approach the question carefully and precisely.

Mass defect

$$^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta$$

Information given:

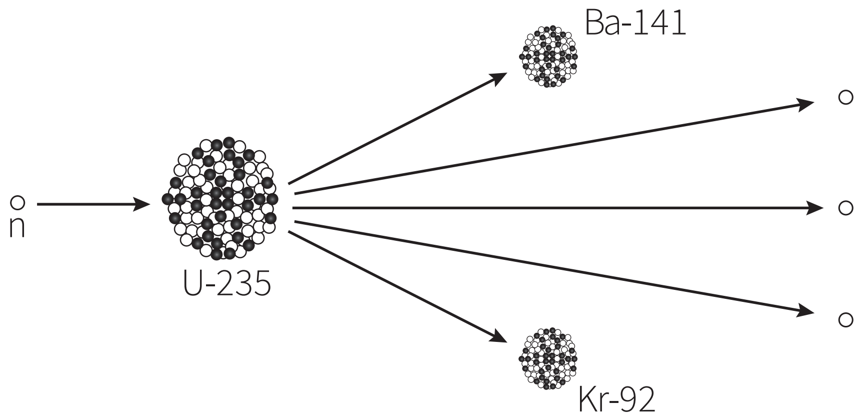
- Atomic mass of carbon-14 $= 14.000\,324\,2\,\text{u}$;
- Atomic mass of nitrogen-14 $= 14.003\,074\,\text{u}$;
- Mass of electron $= 0.000\,549\,\text{u}$;

1.	Find mass on both sides of equation	Mass of LHS = nuclear mass of carbon-14 $14.003242 - 6(0.000549) = 13.999948$ Mass of RHS = nuclear mass of nitrogen-14 + mass of 1 e^- $14.003242 - 7(0.000549) + 0.000549$ $= 13.999780$
2.	Find the mass difference	Mass difference = mass of LHS - mass of RHS $13.999948 - 13.999780 = 0.000168$
3.	Convert to $\dfrac{\text{J}}{\text{mol}}$	Energy released per decay $0.000\,168\,\text{u} \times 931.5\,\text{MeV}\,\text{u}^{-1}$ $= 0.156\,492\,\text{MeV}$ 14 g of carbon-14 means 1 mol (6.02×10^{23}) decays $0.156\,592\,\text{MeV} \times 6.02 \times 10^{23}$ $= 9.424 \times 10^{22}\,\text{MeV}$ $9.424 \times 10^{22}\,\text{MeV} \times 16 \times 10^{-13}$ $= 15.142\,\text{J}$

7.5 Fission, fusion, and antimatter

7.5.1 Fission

Fission the nuclear reaction whereby a heavy nucleus splits into two smaller nuclei of roughly equal mass, elements heavier than iron release of energy



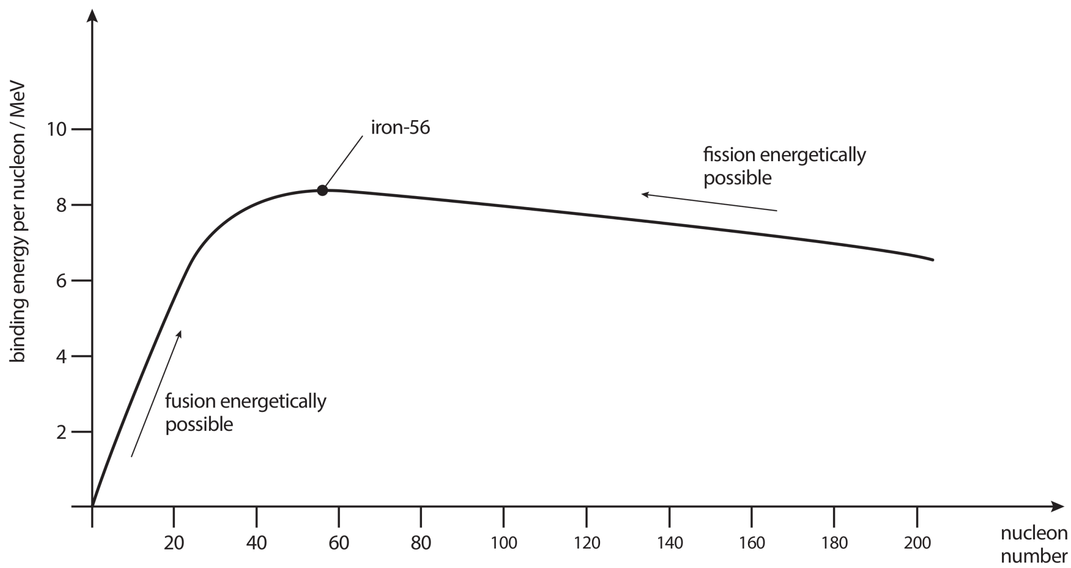
It is the reaction that is used in nuclear reactors and atomic bombs. A typical single reaction might involve bombarding a uranium nucleus with a neutron. This can cause the uranium nucleus to break up into two smaller nuclei. Only one neutron is needed to initiate the reaction while three are released. Each neutron can continue causing another reaction; this is called a chain reaction.

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n + \text{energy}$$

7.5.2 Fusion

Fusion two nuclei 'fuse' to form one larger nuclei, when elements up to iron are formed energy is released in the process. This is the process that fuels stars.

$${}^2_1\mathrm{H} + {}^3_1\mathrm{H} \longrightarrow {}^4_1\mathrm{He} + {}^1_0\mathrm{n} + \text{energy}$$



Whenever a nuclear reaction (fission or fusion) releases energy, the products of the reaction are in a lower energy state than the reactants. Loss of mass must be the source of this energy. In order to compare the energy states of different nuclei, physicists calculate the binding energy per nucleon.

- Binding Energy: energy released when a nuclide is assembled from its individual components. Or, the energy required when the nucleus is separated into its individual components.
- Binding Energy per Nucleon: energy released per nucleon when a nuclide is assembled from its individual components. Or, the energy required per nucleon when the nucleus is separated into its individual components.
- The nucleus with the largest binding energy per nucleon is Iron-56 (shown in the graph above on the highest point). A reaction is energetically feasible if the products of the reaction have a greater binding energy per nucleon than the reactants.

7.5.3 Antimatter

The nuclear model given in the previous pages is somewhat simplified. One important component of the nuclear model that has not been mentioned is antimatter.

Antimatter every form of matter has an equivalent form of antimatter. When matter and antimatter come together, they annihilate each other, this occurs in particle colliders.

Another form of radioactive decay that may take place is υ^+ or positron decay. A proton decays into a neutron and the antimatter version of an electron, a positron, is emitted.

$$
\begin{aligned}
{}^{1}_{1}\mathrm{p} &\longrightarrow {}^{1}_{0}\mathrm{n} + {}^{0}_{+1}\beta^{+} + \upsilon \\
{}^{19}_{10}\mathrm{Ne} &\longrightarrow {}^{19}_{9}\mathrm{F} + {}^{0}_{+1}\beta^{+} + \upsilon
\end{aligned}
$$

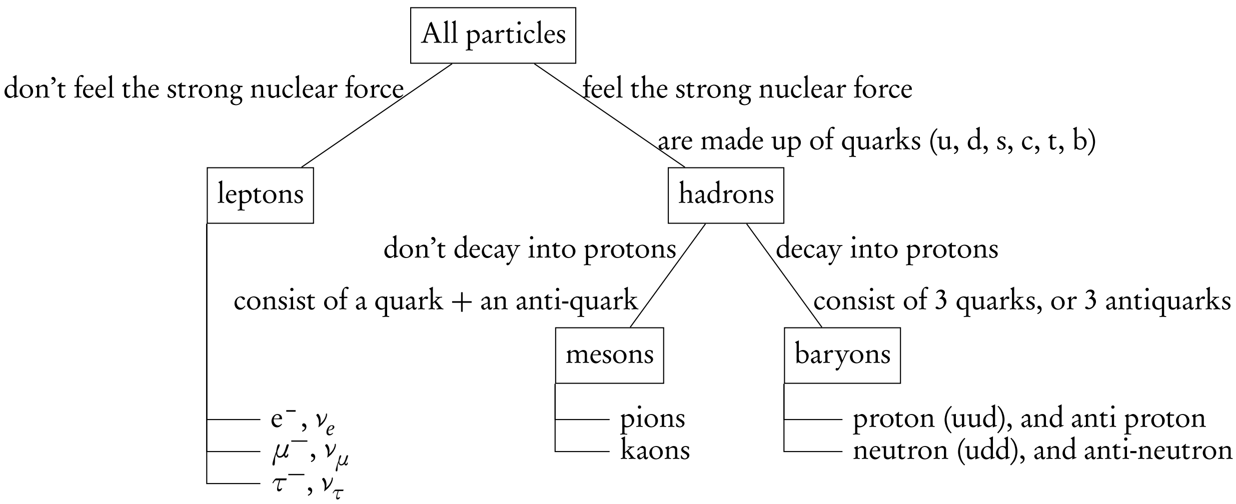
7.6 Elementary particles

7.6.1 Quarks, leptons and exchange particles

Elementary particle is a particle that is not made out of any smaller component particles

Note that protons and neutrons are subatomic, not elementary particles

These are the three classes of elementary particles: quarks, leptons and exchange particles (bosons).



7.6.2 Quarks – strong nuclear forces

Quark flavour		Charge Q	Baryon number B	Strangeness S
Up	u	$+2/3e$	$+1/3$	0
Down	d	$-1/3e$	$+1/3$	0
Strange	s	$-1/3e$	$+1/3$	-1
Charm	c	$+2/3e$	$+1/3$	0
Bottom	b	$-1/3e$	$+1/3$	0
Top	t	$+2/3e$	$+1/3$	0

Each quark has an associated anti-quark which has the same mass, but opposite electric charge and baryon number. So while each quark has baryon number of $+\frac{1}{3}$, and each anti-quark a baryon number of $-\frac{1}{3}$, the baryon number B of the combined particle is the sum of the baryon numbers of the individual quarks.

3 quarks combine to form baryons (B=1)
Proton p^+: uud - TUO-Door = two up one down
Neutron n^0: udd - OUT-Door = one up two down

1 quarks and 1 anti-quark form mesons (B=0)
e.g. **Pion π^+:** u anti-d, π^0 & π^- form from u/anti-u and d/anti-d quarks
Kaons

Isolating a single quark is impossible, it would require infinite energy.

7.6.3 Leptons – weak nuclear forces

Leptons are *not* made of quarks

There are 6 types of leptons: electron and its neutrino, muon and its neutrino, tau and its neutrino. Tau is the heaviest, then muon, the electron is the lightest. Leptons interact with the weak nuclear interactions, and those with electric charge also interact with the electromagnetic interaction.

Leptons are assigned a quantum number called a family lepton number: L_e, L_μ, L_τ. The table below shows the various leptons and their properties. Again, take into account that anti leptons have opposite properties except their mass.

Lepton	L_e	L_μ	L_τ
e^-	+1	0	0
ν_e	+1	0	0
μ^-	0	+1	0
ν_μ	0	+1	0
τ^-	0	0	+1
ν_τ	0	0	+1

7.6.4 Bosons = exchange particles

Summarizing the elementary particles up to now in a table we have 6 quarks, 6 leptons and apparently 4 more exchange particles that represent the interactions or forces between those. For example, electrostatic interactions are mitigated by photons $= \gamma$, the strong nuclear force by gluons $= g$, and the weak nuclear force by $Z^0/W^+/W^-$ exchange particles.

Gravity and the Higgs boson are ignored here

				Exchange particles	Interaction
Quarks	u	c	t	γ	EM
	d	s	b	g	SN
Leptons	e^-	μ^-	τ^-	Z	WN
	ν_e	ν_μ	ν_τ	$W^\pm$	WN

The mass of the exchange particle and the range of the interaction are inversely proportional, so while a photon has no mass it has infinite range. When two electrons are repelled due to their similar charges a (virtual) photon is responsible to carry this repelling interaction. These interactions can be shown in Feynman diagrams.

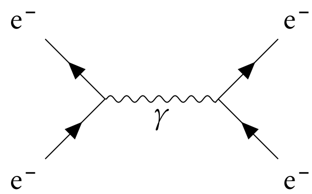
7.6.5 Feynman diagrams

Rules for Feynmann diagrams:

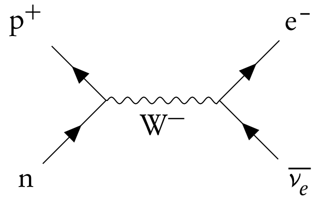
- y-axis: Originating particles start at the bottom, produced particles point upwards
- x-axis: Exchange particles change the nature or direction of the originating particles (so after meeting the horizontal vertex the outgoing particles have changed from the ingoing particles).

To determine whether the interaction (horizontal) is γ, W^- or W^+ we must consult the laws of conservation of charge, baryon number and lepton number. The arriving particles are drawn from the bottom (before), the exiting particles to the top (after).

Two electrons approach



A $n^0 \longrightarrow p^+ + e^+ + \overline{\nu_e}$



		Left	Right	Change	Left	Right	Change
Q	before	−1	−1	-	0	0	
	after	−1	−1	-	+1	−1	$+1-1=0$
B	before	0	0	-	1	0	
	after	0	0	-	1	0	0
L_e	before	+1	+1	-	0	+1	
	after	+1	+1	-	0	−1	$+1-1=0$
Exchange particle		γ (no change)			W^- (to balance charge)		

The change in L_μ and L_τ, which are not shown here, should also be 0 (= conservation)

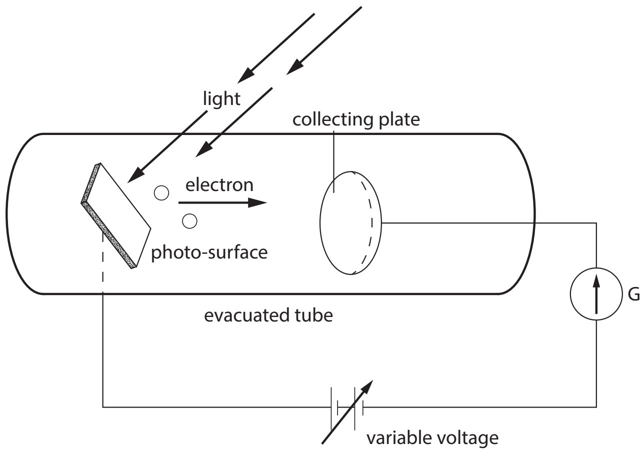
7.6.6 Higgs particle

The theory of quarks, leptons and exchange particles defines the so-called Standard Model of particles and interactions. The centerpiece of this model is the Higgs particle, which is responsible through its interactions for the mass of the particles of the standard model.

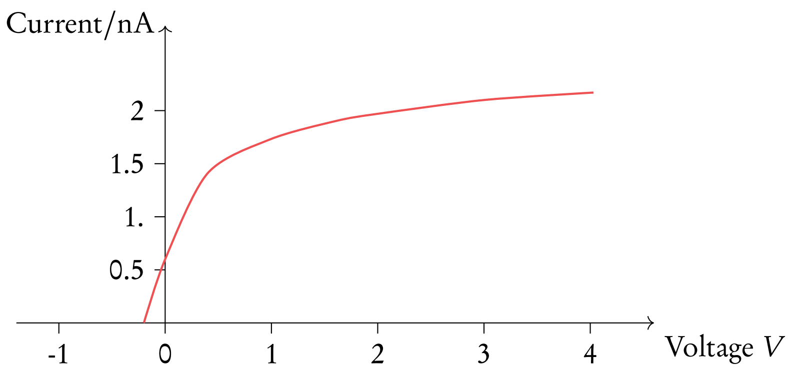
7.7 Interaction of matter with radiation

7.7.1 Photoelectric effect

The photoelectric effect is the phenomenon in which electromagnetic radiation incident on a metallic surface causes electrons to escape from the surface.



The voltage at which the current becomes zero is called the stopping voltage V_s.



Surprising observations

1. The stopping voltage is independent of the intensity of the light source.
2. The electron energy depends on the frequency of the incident light.
3. There is certain minimum frequency below which no electrons are emitted.
4. Electrons are emitted with no time delay.

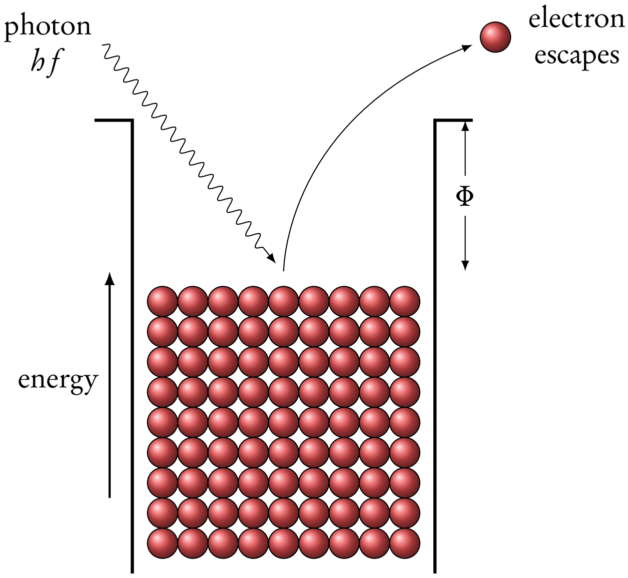
Einstein's Explanation

Einstein suggested light should be thought of as a collection of quanta. Each quantum has energy E given by $E = hf$, where h is Planck's constant and f is the frequency.

The momentum of a photon is

$$P = \frac{E}{C} = \frac{hf}{C} = \frac{h}{\lambda}$$

According to Einstein a single photon of frequency f is absorbed by a single electron in the photo surface, so its energy is hf.



DB

The electron will need an energy Φ to free itself from the metal, additional energy will be transferred as kinetic energy of the electron

$$E_k = hf - \Phi$$

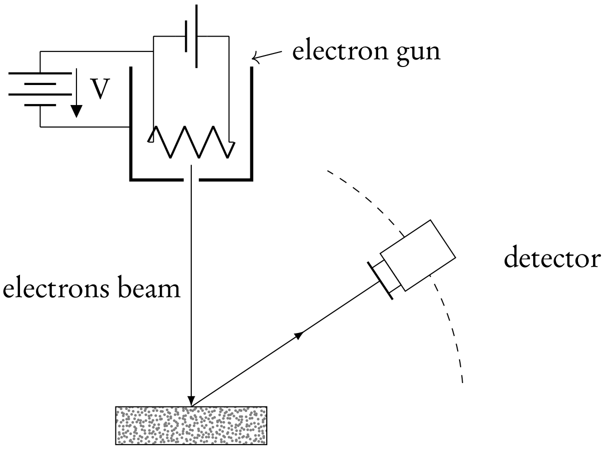
Matter waves

The deBroglie wavelength is defined as:

$$\lambda = \frac{h}{p}$$

All moving particles are assigned a wavelength this is called the duality of matter.

The Davisson Germer Experiment has been used to prove the wave nature of electrons.



The electron energy is low so the top layer of atoms scatters the electrons. The distance d is known and the angle θ so using

$$d \sin\theta = n\lambda$$

We can calculate wavelength λ of the electrons.

7.7.2 Quantisation of angular momentum

The angular momentum of an arbitrary electron is DB

$$mvr = \frac{nh}{2\pi} \qquad \text{(Bohr Condition)}$$

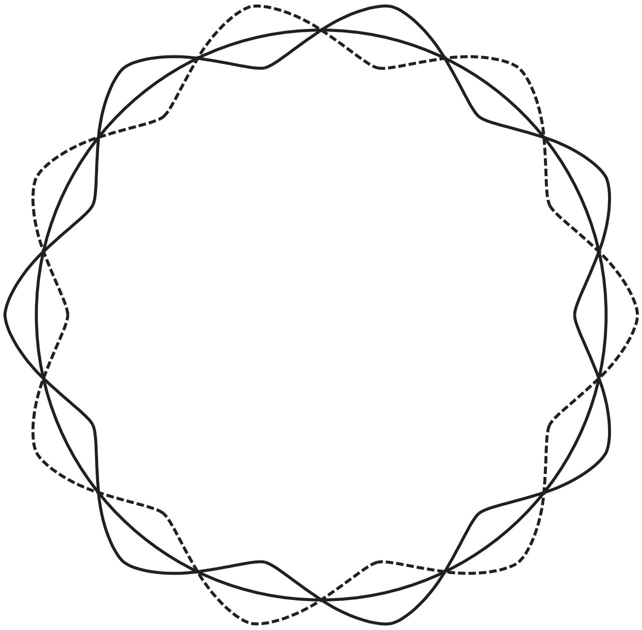
and its kinetic energy

$$K_e = \frac{mv^2}{2} = \frac{n^2h^2}{8\pi^2mr^2}$$

In a hydrogen atom this theory predicts quantised energy levels of the electron given by:

$$E = -\frac{13.6}{n^2}ev$$

The allowed electron orbits are those for which an integral number of electron wavelength fit on the circumference of the orbit.



The wave Function

What kind of waves are we talking about? This question was answered by Austrian physicist Erwin Schrodinger. Schrodinger theory assumes that there is a wave associated with an electron called the wave function:

$$\Psi(x,t)$$

The probability that an electron is found at a particular small volume Δv is given by:

$$P = |\Psi(x,t)|^2 \Delta v$$

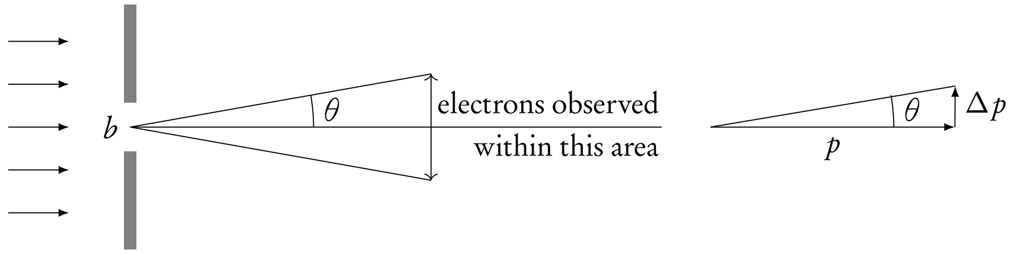
The Uncertainty Principle

> The Heisenberg uncertainty principle applied to position and momentum states that it is not possible to simultaneously measure the position and momentum of a particle with indefinite precision

DB This uncertainty Δx and ΔP are related by

$$\Delta x \Delta P \geq \frac{h}{4\pi}$$

A good example takes single-slit diffraction as the situation, this should provide more explanation of where the uncertainty formula comes from.



The uncertainty in an electron's momentum ΔP is in the vertical direction due to diffraction. We know from diffraction that the diffraction abgle θ is

$$\theta \sim \frac{\lambda}{b}$$

Substituting using the uncertainty in momentum

$$\frac{\lambda}{b} \sim \frac{\Delta P}{P}$$

b is the still width which is actually

$$b = z\Delta x$$

so

$$\frac{\lambda}{z\Delta x} \sim \frac{\Delta P}{P} \Rightarrow \Delta x \Delta P = \frac{\lambda P}{2}$$

Finally using

$$\lambda = \frac{h}{P}$$

$$\Delta x \Delta P = \frac{h}{2}$$

Electron in a Box

Imagine an electron trapped in a box of length L so that $\Delta x = L/2$, thus

$$\Delta P = \frac{h}{4\pi \Delta x} = \frac{h}{2\pi L} \qquad \text{(momentum)}$$

and the kinetic energy of the electron is:

DB

$$E_k = \frac{P^2}{2m} \sim \frac{h^2}{8\pi^2 m L^2}$$

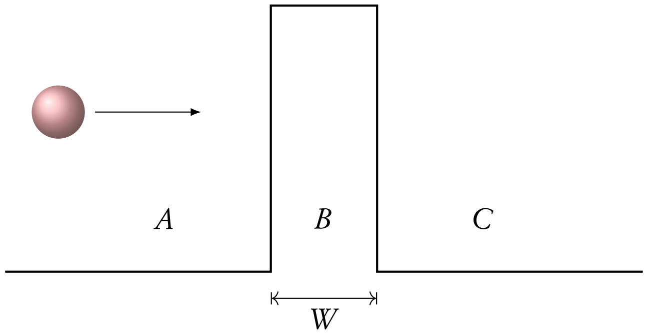
Uncertainty in energy and time

$$\Delta E \Delta t \sim \frac{h}{4\pi}$$

this could apply to decaying particles

Tunneling

Imagine a proton approaching an energy barrier as shown in the image below

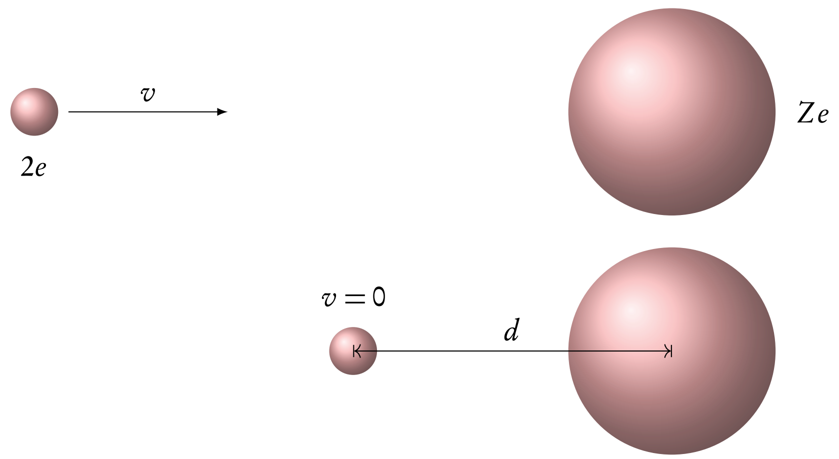


Even if the energy needed to overcome this barrier is greater than the energy of the proton, according to quantum mechanics there is a certain probability for the proton to tunnel through the barrier. Three factors affect the probability of transmission

1. The mass m of the particle.
2. The width of the barrier.
3. The energy difference ΔE between the barrier and the particle.

7.8 Rutherford scattering

We can use simple calculations to calculate the distance of closest approach:



Consider an alpha particle that is projected head-on toward a stationary nucleus Q at its closest point at a distance d.

$$E = k\frac{2Ze^2}{d}$$

thus

$$d = k\frac{2Ze^2}{E_k}$$

When the kinetic energy is large we can measure the radius of the nucleus R.

$$R = R_0 A^{\frac{1}{3}}$$
$$R_0 = 1.2 \times 10^{-15}\ \mathrm{m}$$

Rutherford derived a theoretical formula for the scattering of alpha particles from nuclei.

$$N \sin^4\left(\frac{\theta}{2}\right) = \text{constant}$$

where N is the number of scattered alpha particles. As the alpha particles approach the nucleus at higher energies deviations from the Rutherford model are observed due to the nuclear force.

Nuclear Energy Levels

The fact that energies of alpha and gamma particles released from nuclei are discrete is evidence for the presence of nuclear energy levels

Neutrino

Wolfgang Pauli hypothesised the existence of a third particle in the products of a beta decay in 1933. Since the energy of the electron in beta decay has a range of possible values, it means that a third very light particle must also be produced so that it carries the reminder of the available energy.

Radioactive Decay Law

DB

The law of radioactive decay states that:

$$\frac{dN}{dt} = -\lambda N$$

λ = decay constant
N = number of nuclei present

The decay constant λ is the probability of decay per unit time.

The decay law

$$N = N_0 e^{-\lambda t}$$

N_0 = initial number of nuclei

ACADEMY

ENERGY PRODUCTION

8

8.1 Energy sources

Primary energy all types of unprocessed energy sources

Secondary energy processed or exploited to mechanical work or electrical form

Exercize.

1. Can you give two examples of each?
2. Estimate how much energy can we actually extract from primary energy sources?

Specific energy, E_s is the amount of energy that can be extracted from a unit *mass* of fuel

Energy density, E_D is the amount of energy that can be extracted from a unit *volume* of fuel

Table 8.1: Specific energy or energy density of fossil fuels

Fuel	Specific Energy $E_s/\mathrm{J\,kg^{-1}}$	Energy density $E_D/\mathrm{J\,m^{-3}}$
uranium-235	7.0×10^{13}	1.3×10^{18}
hydrogen	1.4×10^{8}	1.0×10^{7}
natural gas	5.4×10^{7}	3.6×10^{7}
gasoline	4.6×10^{7}	3.4×10^{10}
kerosene	4.3×10^{7}	3.3×10^{10}
diesel	4.6×10^{7}	3.7×10^{10}
coal	3.2×10^{7}	7.2×10^{10}

Renerwable energy a fuel that is created faster or equally fast as it is consumed.

Non-renewable energy a fuel that is consumed faster than it is created.

8.1.1 Fossil fuels

Fossil fuels are produced by the decomposition of buried animal and plant material due to pressure and bacteria

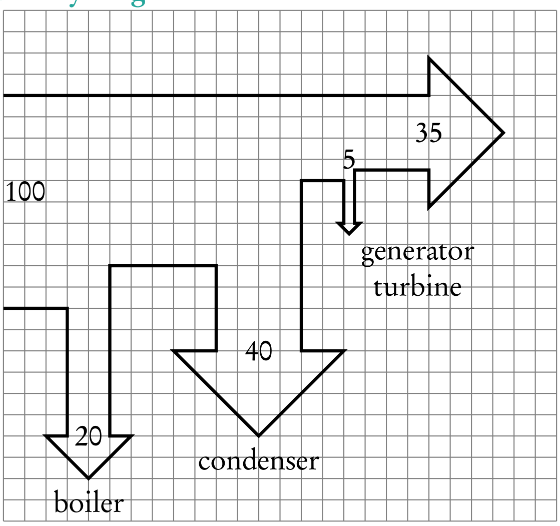
Table 8.2: Energy sources and the percentage of the total energy production for each. The third column gives the mass of carbon dioxide emitted per unit of energy produced from a particular fuel. Fossil fuels account for about 80% of the total energy production.

Fuel	Percentage of total energy production / %	Carbon dioxide emission / $g\,MJ^{-1}$
oil	32	70
natural gas	21	50
coal	27	90
nuclear	6	—
hydroelectric	2	—
biofuels	10	—
oil	<2	—

Given the environmental hazards of coal and oil, we will assume we can only use natural gas in the IB. To represent energy flows, we use a Sankey diagram

Exercize.

Sankey diagram



Given that:

$$\text{efficiency} = \frac{\text{useful power}}{\text{input power}}$$

$$\text{power} = \frac{\text{energy}}{\text{time}}$$

what is the efficiency of our power plant?

Advantages	Disdvantages
Cheap	Fossils fuels will run out
High power output	Environmental hazards
Ease of use	

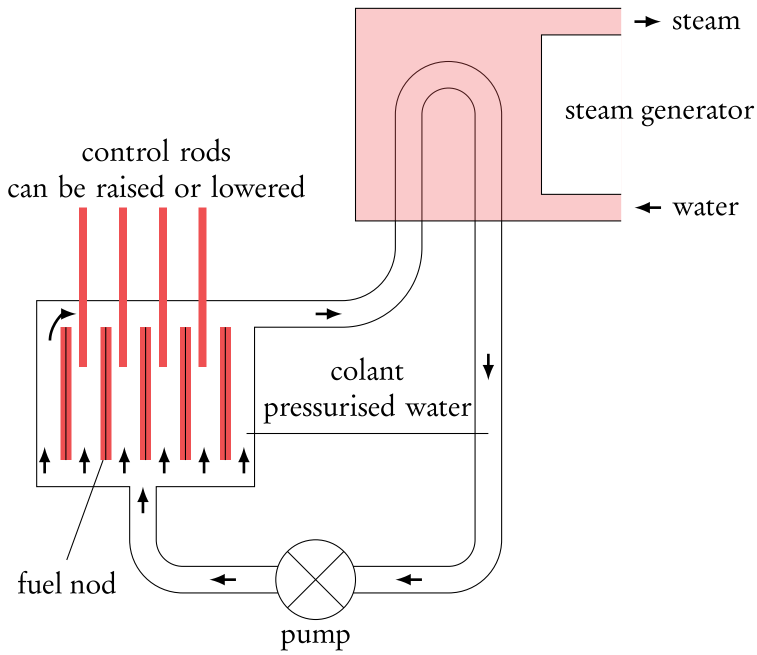
8.1.2 Nuclear power

Nuclear power a nuclear reactor is a machine in which nuclear fission reactions take place producing energy.

The fuel used is uranium-235. Can you remember this reaction from the last topic?

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

Reaction Process



This is a self sustained reaction, because it is a *chain reaction*. For the reaction to get going, we need to have minimum mass called the *critical mass* of uranium. To control the reaction, neutrons are slowed using a *moderator* which is usually a material surrounding the *fuel rods*. The moderator (usually graphite or water) will heat up and this heat may be extracted using a *heat exchanger*. finally, *control rods* may be introduced to absorb neutrons and thus decrease the rate of reaction.

Advantages	Disdvantages
High power output	Disposing of radioactive waste
Large reserves of nuclear fuels	Uranium mining
No greenhouse gases	Risk of nuclear explosion

8.1.3 Solar power

Solar power on a clear day earth receives $\sim 1000\,\mathrm{W}^2\,\mathrm{m}^{-1}$ of energy from the sun. There are two methods for harnessing this energy.

Solar panels collect heat and water in pipes underneath is heated.

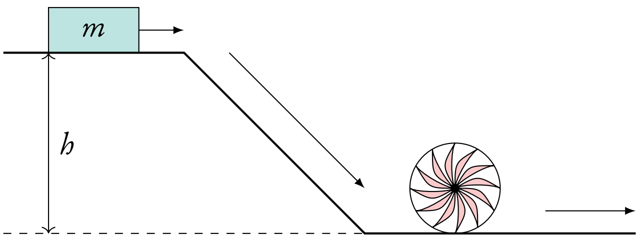
Photovoltaic cells convert light directly into electricity at an efficiency of $\sim 25\%$

8.1.4 Hydroelectric power

Hydroelectric power Power derived from moving masses of water

Derivation

Water falls from a resevoir down to a power station. The water has a mass m and falls a height h.



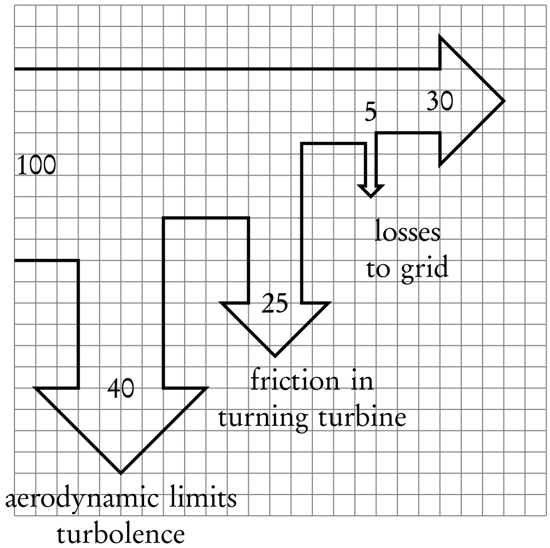
1. The potential energy is mgh
2. $m = \rho\Delta V$, where $\rho =$ density, ΔV is volume.
3. $P = \dfrac{\Delta E}{\Delta t} = \dfrac{mgh}{\Delta t} = \dfrac{\rho\Delta V gh}{\Delta t} = \rho g h \dfrac{\Delta V}{\Delta t}$
4. $Q = \dfrac{\Delta V}{\Delta t}$ (volume flow rate)
5. $P = \rho Q g h$

What is the main problem with hydroelectric power?

8.1.5 Wind power

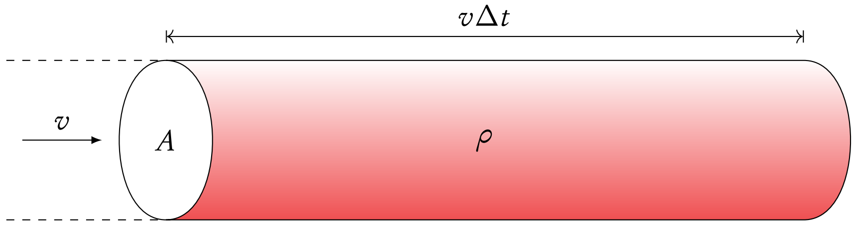
Wind power power derived from moving masses of air

Wind power speaks for itself. Given, we have all seen wind generators at some point. Typically about $\sim 30\%$ of the power carried in wind is converted to energy as shown by Sankey diagram below.



Derivation

Consider the mass of air that can pass through a tube of cross sectional area A with velocity v in time Δt. The air density is ρ



1. $m = \rho A v \Delta t$ (mass of air)

2. Kinetic energy of air is $E = \frac{1}{2}mv^2 = \frac{1}{2}\rho A v \Delta t v^2 = \frac{1}{2}\rho A \Delta t v^3$

3. Power $= \frac{E}{\Delta t} = \frac{1}{2}\rho A v^3$

Energy source	Advantages	Disdvantages
Solar	Cheap Renewable Clean	Low power output Inconsistent
Hydroelectric	Cheap Clean	Water storage Damage to local ecology
Wind	Clean Renewable	Large infrastructure Inconsistent

8.2 Thermal energy transfer

Heat can be transferred by three different methods.

Conduction is the transfer of energy due to high energy electrons colliding with neighbouring molecules.

Convection is is best explained by an example.

Air over a hot radiator in a room is heated, expands and rises, transferring warm air to the rest of the room. Cold air takes its place through convection currents and the process repeats.

Radiation is the transfer of energy through electromagnetic radiation.

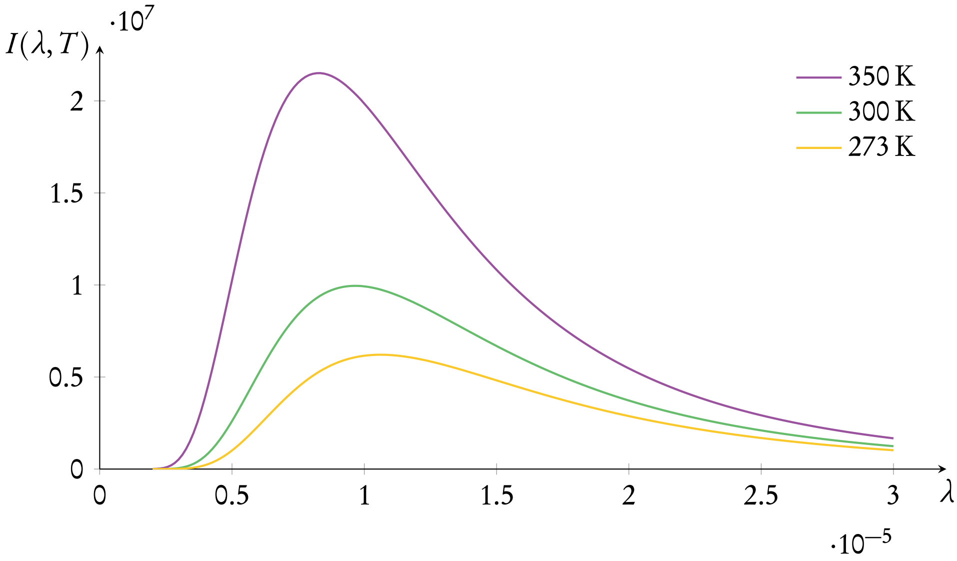
Black-body radiation The power radiated by a body is governed by Stefan-Boltzman law

$$P = \varepsilon \sigma A T^4$$

- P = power $[\mathrm{J\,s^{-1}}]$
- ε = emissivity
- σ = Stefan-Boltzmann constant $5.67 \times 10^{-8}\ \mathrm{W m^{-2} K^{-4}}$
- A = area $[\mathrm{m^2}]$
- T = thermodynamic temperature $[\mathrm{K}]$

Emissivity ε measures how effectively a black-body radiates.
$\varepsilon = 1 \Rightarrow$ black body.

Figure 8.1



Most energy is radiated at a specific wavelength λ_{max} determined by Wien's displacement law. For green box,

$$\lambda_{max} T = 2.90 \times 10^{-3}\,\text{km}$$

This is shown for various temperatures in the figure below.

Solar Constant Intensity is the power of radiation received per unit area.

$$I = \frac{P}{4\pi r^2 \sim 1400\,\text{W m}^{-2}}$$

This is the solar constant S and is the amount of solar radiation at the top of earth's atmosphere.

$$\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$$

Using the albedo we can find the average intensity incident on the earth.

$$I_{avg} = \frac{(1-\alpha)S}{4} = 245\,\text{W m}^{-2}$$

Temperature at earth's surface

We are interested in average temperature at the Earth's surface. The earth radiates power from the entire surface area of its spherical shape, so the power radiated is

$$P_{\text{out}} = \sigma A T^4$$

We are assuming earth to be black body so, $\varepsilon = 1$.

$$I_{\text{out}} = \frac{P_{\text{out}}}{A} = \sigma T^4$$

Equating the incident and outgoing intensities we get

$$\frac{(1-\alpha)S}{4} = \sigma T^4$$

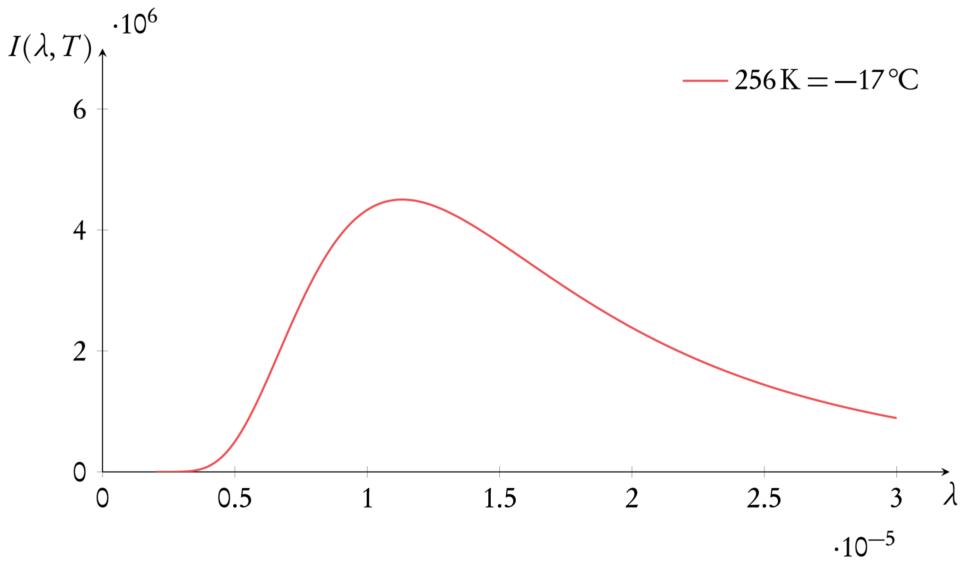
Solving the equation, we get

$$T = \sqrt[4]{\frac{(1-\alpha)S}{4\sigma}}$$

This evaluates to

$$T = \sqrt[4]{\frac{245}{5.67 \times 10^{-8}}} = 256\,\text{K}$$

This temperature is $-17\,°\text{C}$.



Greenhouse effect Greenhouse gases strongly absorb infrared radiation from the atmosphere, when re-radiated in all directions, these gases account for additional warming. These gases include Water vapour, Carbon dioxide, Methane and Nitrous oxide.

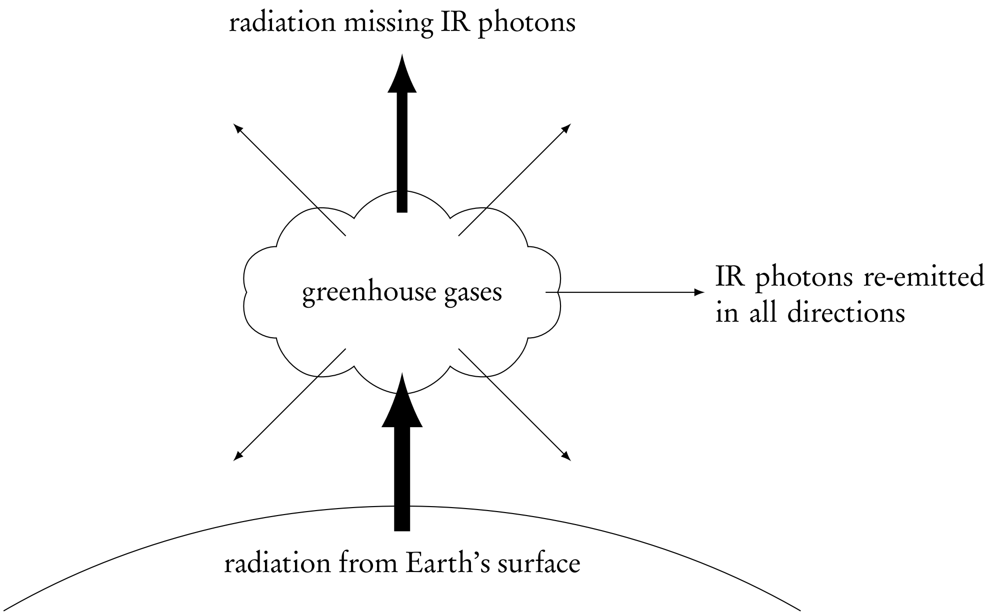


Table 8.3: Sources of greenhouse gases

Greenhouse gas	Natural Sources	Anthropogenic sources
H_2O	evaporation of water from oceans, rivers and lakes	irrigation
CO_2	forest fires, volcanic eruptions, evaporation of water from oceans	burning fossil fuels in power plants and cars, burning forests
CH_4	wetlands, oceans, lakes and rivers, termites	flooded rice fields, farm animals, procession of coal, natural gas and oil, burning biomass
N_2O	forests, oceans, soil and grasslands	burning fossil fuels, manufacture of cement, fertilizers, deforestation (reduction of nitrogen fixation in plants)